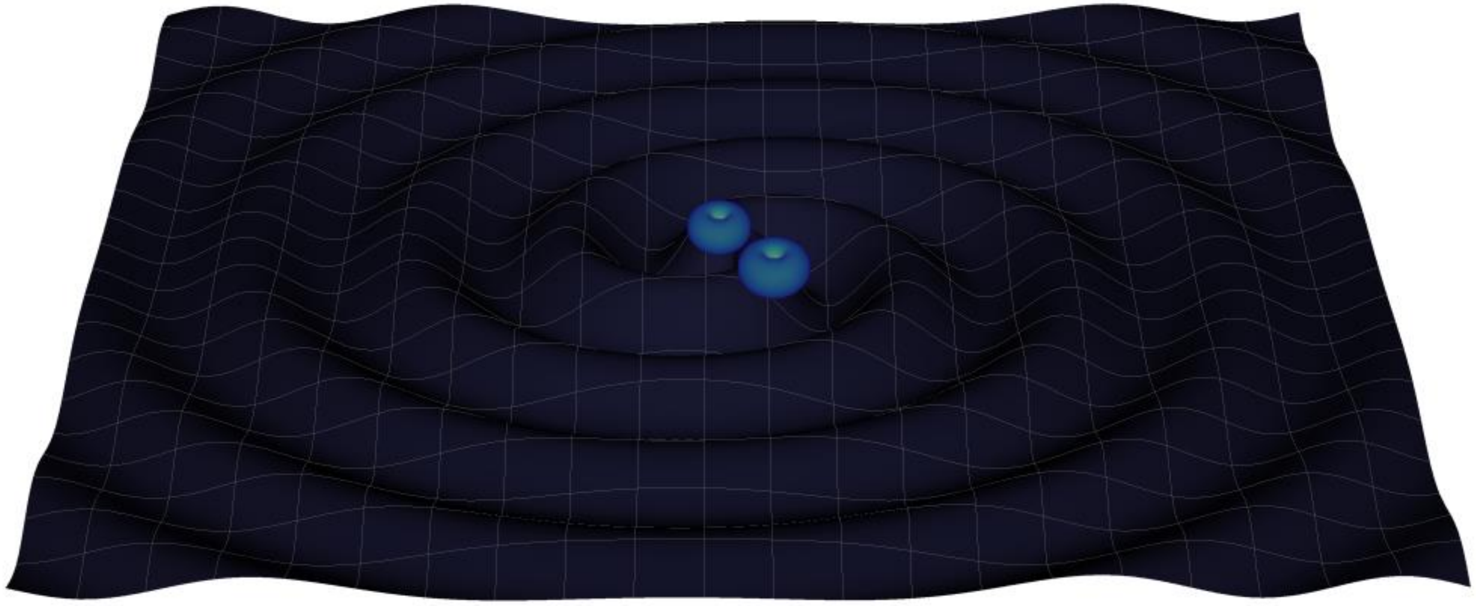
Constraining fundamental properties of boson stars through their multipole moments



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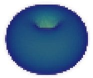
Motivations for the work

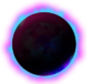
- Gravitational waves allow to probe the nature of compact objects and to search for new physics

Exotic Compact Objects

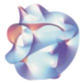


Boson stars

Proca stars



BHs with scalar hair



Fuzzballs

. . .

- The multipolar structure affects the dynamics of binary systems and their gravitational wave emission

Stationary axysimmetric spacetime $\Rightarrow$ scalar mass moments $M_0, M_2 \ldots$ and current moments $S_1, S_3 \ldots$

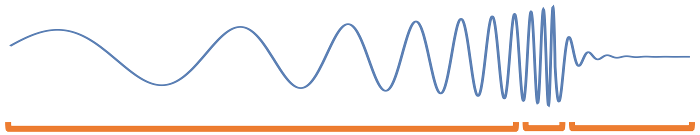
for a Kerr black hole

$$\boxed{M_l + iS_l = M^{l+1}(i\chi)^l} \qquad \chi = \frac{J}{M^2}, \quad M = M_0$$

Not true for a generic compact object!

E.g. $h \sim \mathcal{A}(f)e^{i(\psi_{BH}(f)+finite\ size\ corr\ )}$

PN expanded waveform



Rotating Boson Stars (BSs)

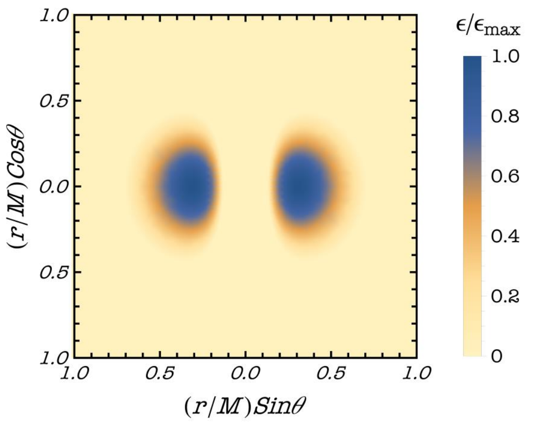
- Boson stars are solutions of the Einstein gravity, minimally coupled to a complex scalar field:

$$L = -\frac{1}{2} g^{\mu\nu} \phi^*_{,\mu} \phi_{,\nu} - \frac{1}{2} V(|\phi|^2)$$

The equations take the form: $G_{ab} = 8\pi T_{ab}$ $\quad \frac{1}{\sqrt{-g}} \partial_a \left(\sqrt{-g} g^{ab} \partial_b \phi \right) = \frac{dV(|\phi|^2)}{d|\phi|^2} \phi$

- *Neutron Stars: Equation Of State* → *Boson Stars: Self-interactions in* $V(|\phi|^2)$

- Mini BSs $\quad V(|\phi|^2) = m^2 |\phi|^2 \quad M_{max} \sim \frac{M_p^2}{m}$
- Massive BSs $\quad V(|\phi|^2) = m^2 |\phi|^2 + \lambda |\phi|^4 \quad M_{max} \sim \frac{M_p^3}{m^2} \lambda^{\frac{1}{2}}$
- Solitonic BSs $\quad V(|\phi|)^2 = m^2 |\phi|^2 \left(1 - \frac{2|\phi|^2}{\sigma^2} \right) \quad M_{max} \sim \frac{M_p^4}{m\sigma^2}$



Normalized energy-density of a BS in a transversal section

Theoretical set up

- Our case is: $V(|\phi|^2) = \frac{1}{2}m^2|\phi|^2 + \frac{1}{4}\lambda|\phi|^4$ in the large self-coupling limit $(\lambda \gg m^2)$

- In the $\lambda \gg m^2$ a reasonable assumption is : $\phi_{,r} \sim 0 \quad \phi_{,\theta} \sim 0 \quad \phi_{,\varphi} \neq 0 \quad \Rightarrow$

1) The stress energy tensor assume the form:

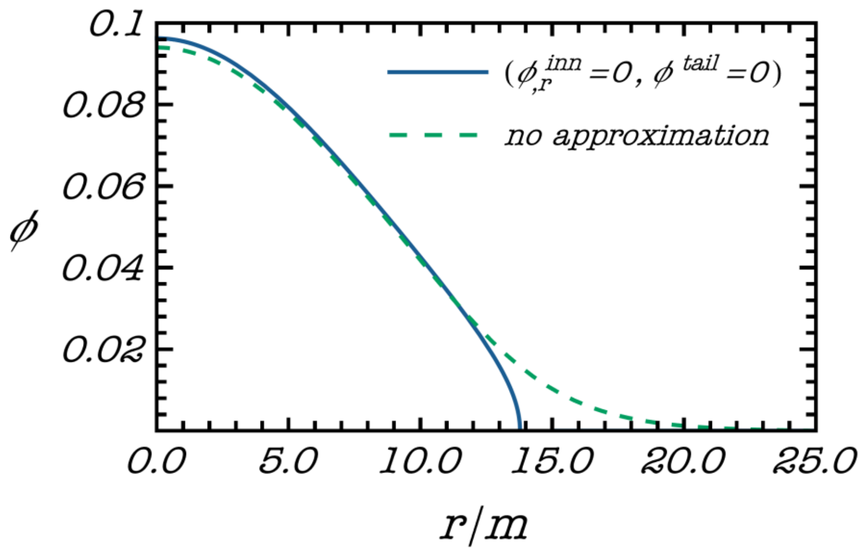
$$T_{\mu\nu} = (\epsilon + P)u_\mu u_\nu + P g_{\mu\nu}$$

2) The scalar field eq. becomes trivial:

$$|\phi|^2 = Max[0, \lambda^{-1}(-g^{tt}\Omega^2 + 2g^{t\varphi}\Omega s - g^{\varphi\varphi} - m^2)]$$

3) The coupling constants can be removed:

$$\lambda^{\frac{1}{2}} / m^2 \equiv M_B$$



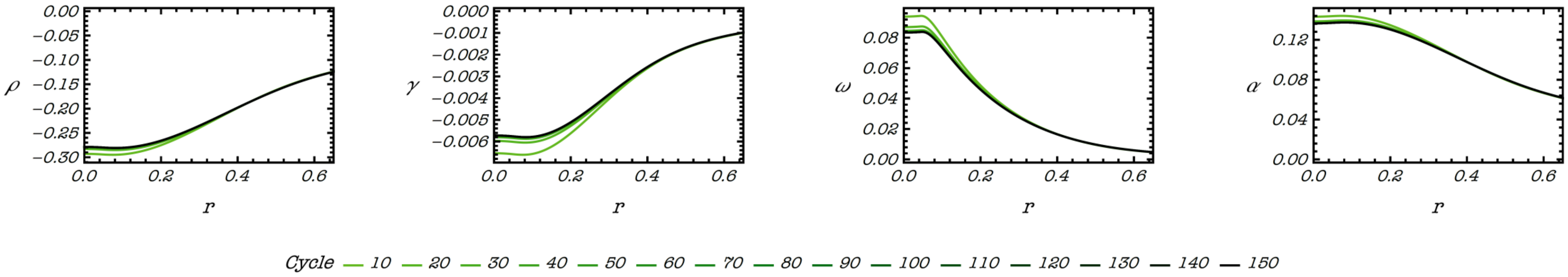
In the figure a similar approximation is considered for the non-spinning case.

Self-consistent field method

- In these conditions, one can apply a self-consistent field method to BSs.

F. D. Ryan, Phys. Rev. D 55, 6081 (1997), ***Spinning boson stars with large self-interaction.***

- $ds^2 = -e^{\gamma(r,\theta)+\rho(r,\theta)}dt^2 + e^{2\alpha(r,\theta)}(dr^2 + r^2 d\theta^2) + e^{\gamma(r,\theta)-\rho(r,\theta)} r^2 \sin\theta^2\, (d\varphi - \omega(r,\theta)dt)^2$



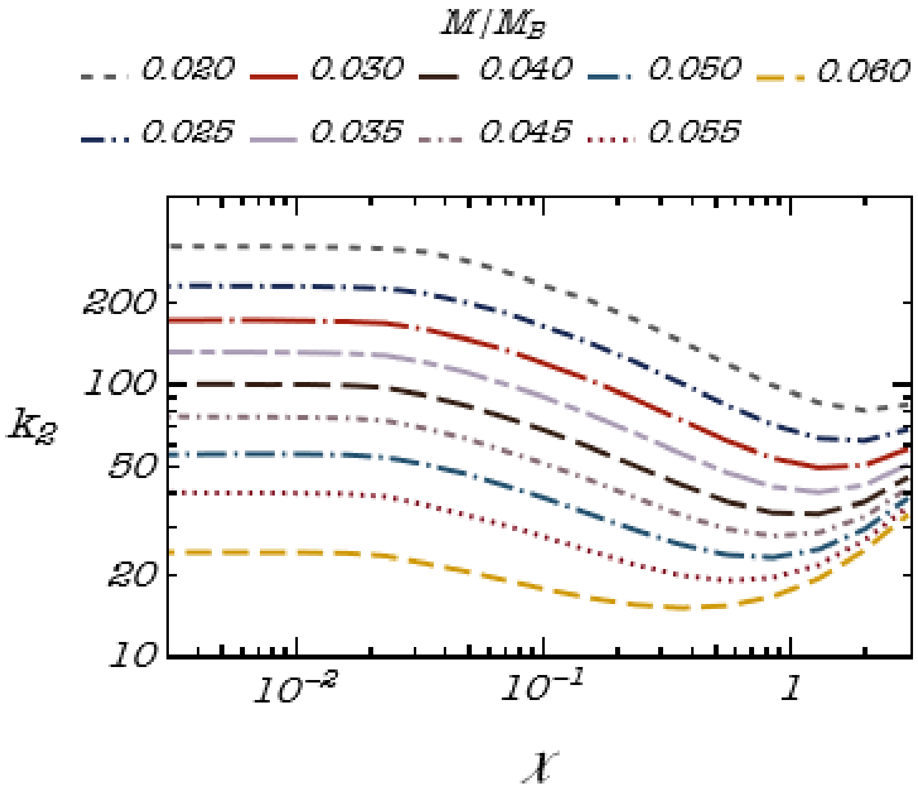
- Mass and current moments $\{M_0, M_2 \ldots\}$, $\{S_1, S_3 \ldots\}$ can be read off:

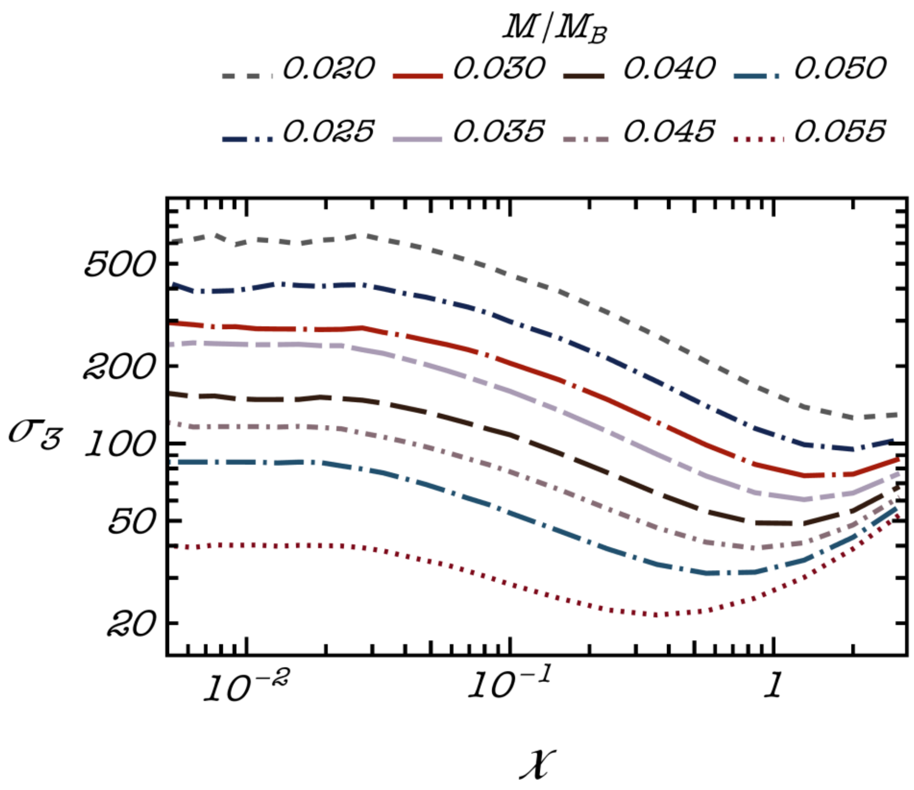
$$\rho(r,\mu) = \sum_{n=0}^{\infty} -2\frac{M_{2n}}{r^{2n+1}} P_2n(\mu) + \text{higher orders}$$

$$\omega(r,\mu) = \sum_{n=1}^{\infty} -\frac{2}{2n-1}\frac{S_{2n-1}}{r^{2n+1}}\frac{P^1_{2n-1}(\mu)}{\sin\theta} + \text{higher orders}$$

Mass-quadrupole and spin-octupole moment

- The plot shows $\kappa_2 = -\frac{M_2}{\chi^2 M^3}$ and $\sigma_3 = -\frac{S_3}{\chi^3 M^4}$ as a function of the dimensionless spin χ for different BS masses (for a Kerr BH $\kappa_2 = \sigma_3 = 1$).

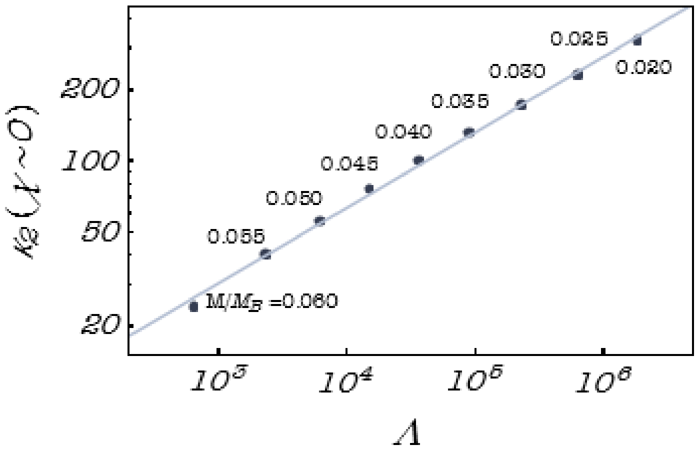


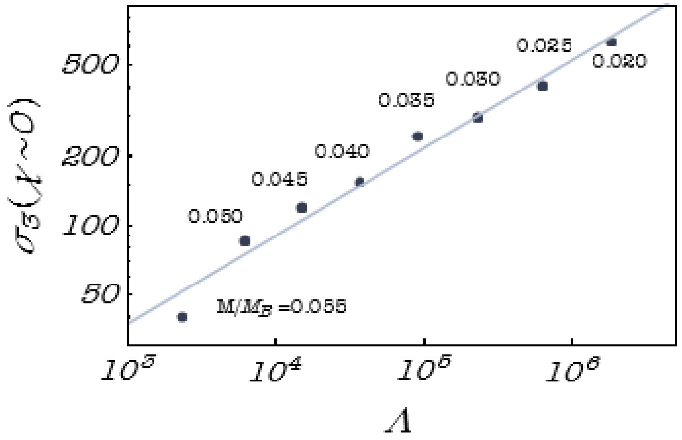


The multipolar structure of fast rotating boson stars: *Massimo Vaglio, Costantino Pacilio, Andrea Maselli, Paolo Pani*, arXiv:2203.07442 (2022)

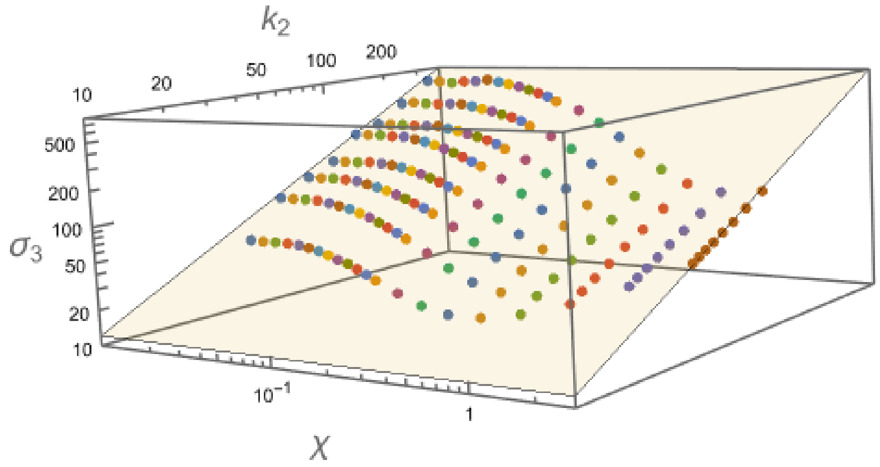
Universal Relations for Boson Stars?

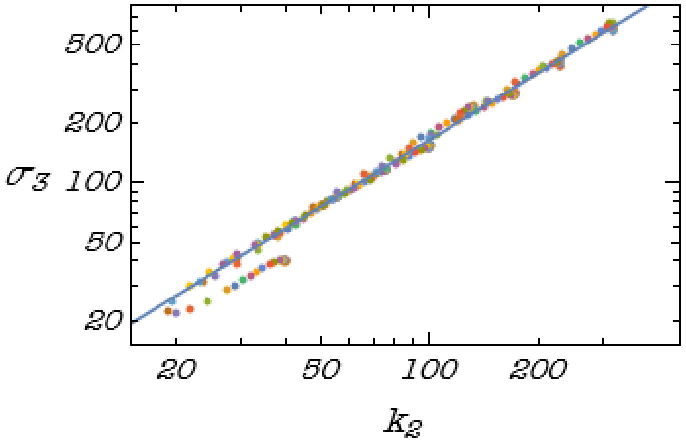
- The reduced quadrupole and octupole moments are simply connected to the tidal deformability





- The relation between κ_2 and σ_3 appears remarkably to be independent on the spin χ





Conclusions and perspectives

We have analyzed the characteristics and the multipolar structure of rotating BSs with quartic self-interactions in the strong coupling limit $(\lambda \gg m^2)$:

- The values of the quadrupole and octupole moments agree with previous results for high spins. In the low spin region, we confirmed the relations $M_2 \sim \chi^2$ ($S_3 \sim \chi^3$), but with improved accuracy, we found absolute values which are larger by a factor of $\sim 2(\sim 3)$.

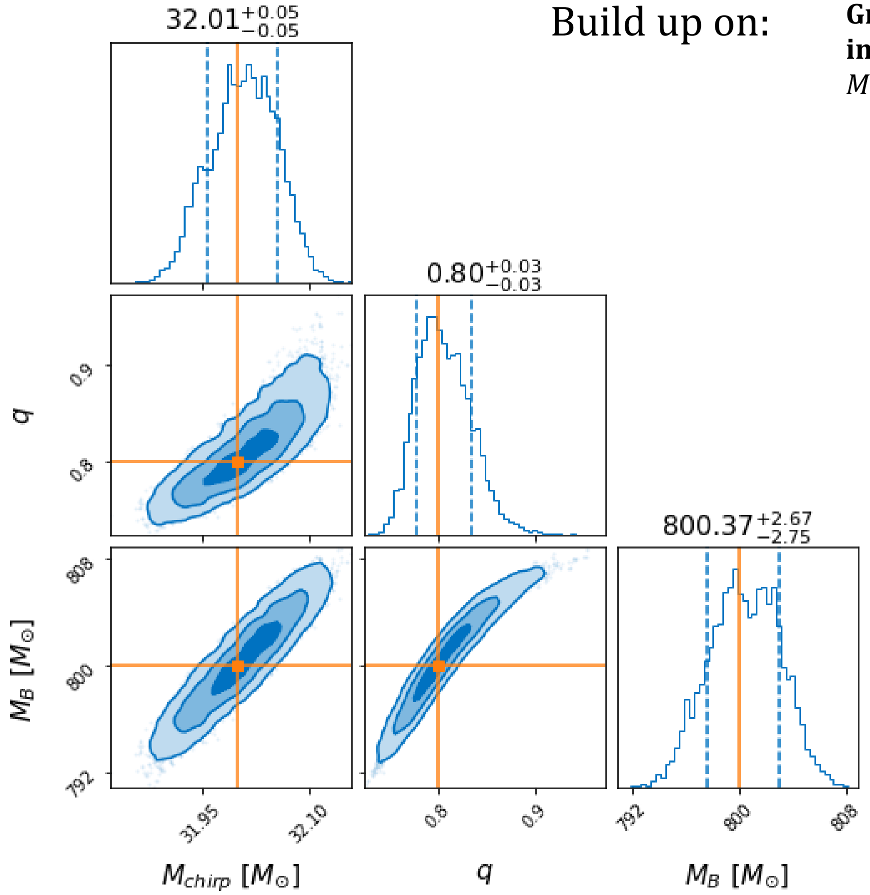
- We found that simple relations hold linking the multipole moments and the tidal deformability.

Future perspectives and extentions of this work include:

- Study the impact of different self-interactions on the multipolar structure to investigate:
  - the universality degree of the relations between multipole moments and tidal deformability
  - the possibility of distinguish the gravitational signal from different BS source models.

Ongoing work

- Build a coherent waveform model which consistently includes different contributions of the multipole moments and assess their detectability through current/future GW observations



Build up on: **Gravitational-wave detectors as particle-physics laboratories: Constraining scalar interactions with a coherent inspiral model of boson star binaries**, *Costantino Pacilio, Massimo Vaglio, Andrea Maselli, Paolo Pani. Phys.Rev. D* 102 (2020) 8, 083002

TaylorF2 coherent waveform with:

$$Q = -\kappa(\chi, M/M_B)\chi^2 M^3$$

and Λ from:

$$\frac{M}{M_B} = \frac{\sqrt{2}}{8\sqrt{\pi}}\left[-0.828 + \frac{20.99}{\log\Lambda} - \frac{99.1}{(\log\Lambda)^2} + \frac{149.7}{(\log\Lambda)^3}\right]$$

N. Sennett et al., Phys. Rev. D, 96, 2 (2017) 024002

Tidal deformability and quadrupoles are funtions of mass, spin and M_B!