The primordial power spectrum in the modified loop quantum cosmology

BaoFei Li

Department of Physics & Astronomy
Louisiana State University

Department of Physics
Zhejiang University of technology

Based on arXiv: 2206.12434 (2022) with Parampreet Singh and Phys. Rev. D101, 086004(2020) with Parampreet Singh and Anzhong Wang and Phys. Rev. D102, 126025(2020) with Javier Olmedo, Parampreet Singh, Anzhong Wang

July 5, 2022

Outline

Main contents:

- Modified LQC models:

  1. Background dynamics: similarities and distinctions
  2. Primordial power spectra: dressed metric vs hybrid

Motivations:

- 1. What are the robust features when quantizing the cosmological sector using a bottom-up approach?
- 2. Can different quantization prescriptions result in distinctive observational signals?

Modified LQC

- In loop quantum cosmology, the big bang singularity is generically resolved and replaced with a quantum bounce.

  [Ashtekar, Pawlowski, Singh, PRL96, 141301; PRD73,124038; PRD74,084003(2006)]

- The classical Hamiltonian in the Ashtekar-Barbero variables consists of a Lorentzian term and a Euclidean term.
- Treat the Lorentzian term in the same way as the Euclidean term $\Rightarrow$ standard LQC. [Ashtekar, et al, Adv. Theor. Math. Phys. 7:233-268, 2003]
- Treat the Lorentzian term separately with different regularizations $\Rightarrow$ mLQC-I and mLQC-II. [Dr. Ma's talk; Dr. Singh's talk; Yang, et al, PLB682 (2009) 1; Dapor and Liegener, PLB785 (2018) 506]
- Differences and similarities between LQC, mLQC-I and mLQC-II:

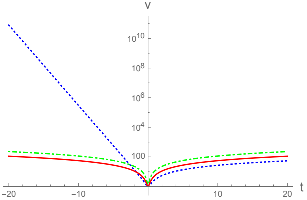
  Background dynamics + Linear order perturbations

Background dynamics: An example

- A massless scalar field minimally coupled to gravity

  [Li, Singh, AW, PRD98, 066016 (2018)]

- LQC mLQC-I mLQC-II



- Qualitative similarities:
  1. bounce happens at a Planck-scale energy density (independent of the initial conditions)
  2. GR is recovered in the expanding phase

- Qualitative differences: (in the contracting phase of mLQC-I)
  1. Emergent De Sitter phase with a Planck-scale Λ
  2. Rescaled Newton's constant [Li, Singh, AW, PRD97, 084029 (2018)]
  3. A negative potential can not guarantee a cyclic universe in mLQC-I [Li, Singh, PRD105, 046013 (2022) and also previous talk by Prof. P. Singh.]

Primordial power spectra: the dressed metric approach v.s. the hybrid approach

- Detailed explanation of the two approaches can be found in [Agullo, Ashtekar, Nelson, PRL109, 251301 (2012); PRD87,043507 (2013); Class. Quant. Grav. 30,085014 (2013),Gomar, et al, PRD90,064015 (2014); JCAP 1506, 045 (2015); PRD96, 103528 (2017)]
- A detailed comparison between two approaches at both the classical and the effective dynamics level can be found in [Li and Singh, arXiv:2206.12434]
- Similarities of two approaches

  1. Based on the classical formulation of the perturbation theory with loop quantization of the background dynamics and Fock quantization of the linear perturbations.

  The dressed metric approach is based on the work by Langlois [Langlois, Class. Quant. Grav. 11,389 (1994)] while the hybrid approach on the seminal work by Halliwell and Hawking [PRD31, 1777(1985)] .

Primordial power spectra: the dressed metric approach v.s. the hybrid approach

2. The backreaction of the linear perturbations is ignored and we make use of the test-field approximation so that the effective dynamics in LQC is valid.

- Differences of two approaches

  1. Fock quantization of the linear perturbations is implemented with respect to different gauge invariant variables:

  The dressed metric approach: Mukhanov-Sasaki variable $\mathrm{Q}_{\vec{\mathrm{k}}}$

  The hybrid approach: the rescaled variable $\nu_{\vec{\mathrm{k}}} = \mathrm{a}\mathrm{Q}_{\vec{\mathrm{k}}}$

  2.Different polymerization of $1/\pi_{\mathrm{a}}$ (conjugate momentum of the scale factor) in the classical mass function which appears in the Mukhanov-Sasaki equation

$$\nu_{\mathrm{k}}'' + \left(\mathrm{k}^2 + \mathrm{m}^2\right)\nu_{\mathrm{k}} = 0. \tag{1}$$

Primordial power spectra: the classical mass functions

- In the dressed metric approach, the classical mass function takes the form

$$\mathrm{m}_{\mathrm{d}}^2 = \frac{3\kappa\bar{\pi}_\phi^2}{\mathrm{a}^4} - 18\frac{\bar{\pi}_\phi^4}{\pi_\mathrm{a}^2\mathrm{a}^6} - 12\mathrm{a}\frac{\bar{\pi}_\phi \mathrm{U}_{,\bar{\phi}}}{\pi_\mathrm{a}} + \mathrm{a}^2\mathrm{U}_{,\bar{\phi}\bar{\phi}} - \frac{\mathrm{a}''}{\mathrm{a}}. \qquad (2)$$

- In the hybrid approach, the classical mass function takes the form

$$\mathrm{m}_{\mathrm{h}}^2 = -\frac{27\bar{\pi}_\phi^4}{2\pi_\mathrm{a}^2\mathrm{a}^6} + \frac{5\kappa\bar{\pi}_\phi^2}{2\mathrm{a}^4} + \frac{9\bar{\pi}_\phi^2\mathrm{U}}{\pi_\mathrm{a}^2} - 12\mathrm{a}\mathrm{U}_{,\bar{\phi}}\frac{\bar{\pi}_\phi}{\pi_\mathrm{a}} - \frac{\kappa^2\pi_\mathrm{a}^2}{72\mathrm{a}^2} + \mathrm{a}^2\mathrm{U}_{,\bar{\phi}\bar{\phi}} - \frac{\kappa}{2}\mathrm{a}^2\mathrm{U}. \qquad (3)$$

- Two mass functions are equivalent on the physical solutions of the classical Friedmann dynamics.

The primordial power spectra: the effective description

- In LQC, one has to polymerize $1/\pi_{\rm a}^2$ and $1/\pi_{\rm a}$ in the classical mass functions in a way consistent with the polymerization of the background dynamics.
- The effective mass function in the dressed metric approach takes the form

$$\mathrm{m}_{\rm eff}^2 = -\frac{4\pi \mathrm{G}}{3}\mathrm{a}^2\rho\left(1+2\frac{\rho}{\rho_{\rm c}}\right)+4\pi\mathrm{G}\mathrm{a}^2\mathrm{P}\left(1-2\frac{\rho}{\rho_{\rm c}}\right)+\mathfrak{U}, \quad (4)$$

- The effective mass function in the hybrid approach reads

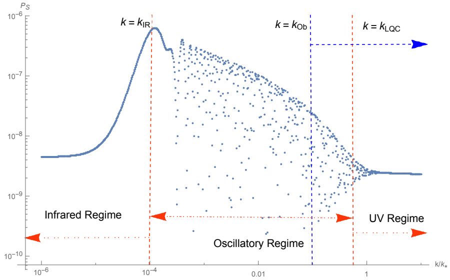
$$\tilde{\mathrm{m}}_{\rm eff}^2 = -\frac{4\pi\mathrm{G}}{3}\mathrm{a}^2\left(\rho-3\mathrm{P}\right)+\mathfrak{U}, \quad (5)$$

with

$$\mathfrak{U} = \mathrm{a}^2\left(\mathrm{U}_{,\bar{\phi}\bar{\phi}}+48\pi\mathrm{G}\mathrm{U}+6\mathrm{H}\frac{\dot{\bar{\phi}}}{\rho}\mathrm{U}_{,\bar{\phi}}-\frac{48\pi\mathrm{G}}{\rho}\mathrm{U}^2\right). \quad (6)$$

and U stands for the potential of the scalar field.

The primordial scalar power spectrum (numerics): general features



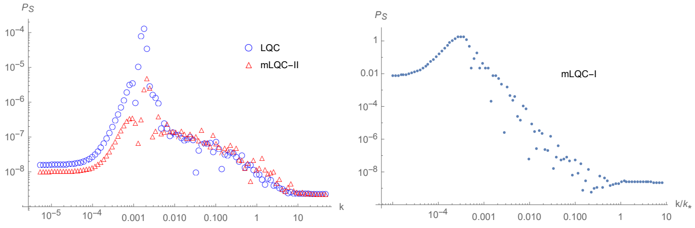
A representative example of the primordial scalar power spectrum in LQC with ϕ^2 potential:

the characteristic wavenumber $\mathrm{k_{LQC}} = 3.20$

the smallest observable wavenumber $\mathrm{k_{0b}} = 0.89(\phi_B = 1.15)$

the window $\mathrm{k} \in (\mathrm{k_{0b}}, \mathrm{k_{LQC}})$ determines the observable modes that can be affected by the QG effects

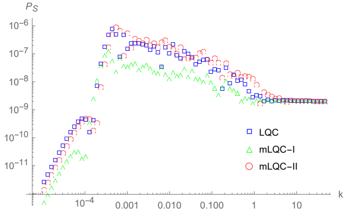
The primordial power spectra: in the dressed metric approach



The primordial scalar power spectra in dressed metric approach [Li, Singh, AW, PRD101, 086004 (2020)]

- Agreement in the UV regime ——- scale-invariant power spectrum
- Less than 40% relative difference in IR (left panel) between LQC and mLQC-II
- Relative difference up to 100% in the intermediate regime between LQC and mLQC-II
- Planck-scale magnitude in the IR regime in mLQC-I

The primordial power spectra: in the hybrid approach

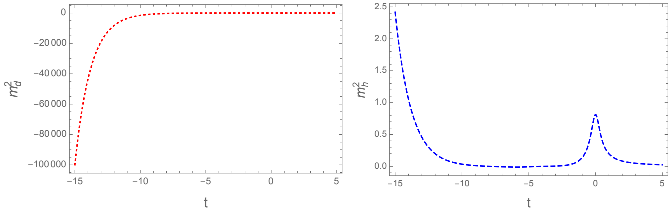


The primordial scalar power spectra in the hybrid approach

[Li, Olmedo, Singh, AW, PRD102, 126025 (2020); Li, Singh, AW, Front. Astron. Space Sci. 8, 701417 (2021)]

- Agreement in the UV regime —— scale-invariant power spectrum
- $\lesssim 50\%$ relative difference in the intermediate and IR regime between LQC and mQLC-II
- Suppressed power spectra in the IR regime of mLQC-I
- $\lesssim 100\%$ relative difference in the intermediate and IR regime between mLQC-I and LQC (mLQC-II)

Why huge differences in the IR regime of mLQC-I between the dressed metric and the hybrid approach



In the left panel: the effective mass in mLQC-I in the hybrid approach
In the right panel: the effective mass in mLQC-I in the dressed metric approach

- In the dressed metric approach, $m_d^2 \approx -\frac{2}{\eta^2}$ with $\eta = -1/(aH)$, so the initial states are chosen to be the BD vacuum

$$v_s(k) = \frac{e^{-ik\eta}}{\sqrt{2k}}\left(1 - \frac{i}{k\eta}\right). \tag{7}$$

- In the hybrid approach, m_h^2 is positive, the initial states are chosen to be second-order (or fourth-order) adiabatic states.

Conclusions

- The background dynamics is attached to the particular regularization used to quantize the Hamiltonian constraint
- The power spectrum in the UV regime agrees with the observations in all three models with both the dressed metric and the hybrid approach
- IR regime in mLQC-I: different behavior of the effective mass functions in two approaches $\rightarrow$ different initial states chosen in the contracting phase $\rightarrow$ Planck-scale magnitude in the dressed metric approach v.s. suppressed magnitude in the hybrid approach

Thank You!