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based on Phys.Rev.D 102 (2020) 8, 085004, Phys.Rev.Lett. 127 (2021) 23, 231301, and Phys.Rev.D 104 (2021), 025009,
with S. Hollands and J. Zahn

Quantum effects in black hole interiors

July 5, 2022

Christiane Klein

Outline

Motivation

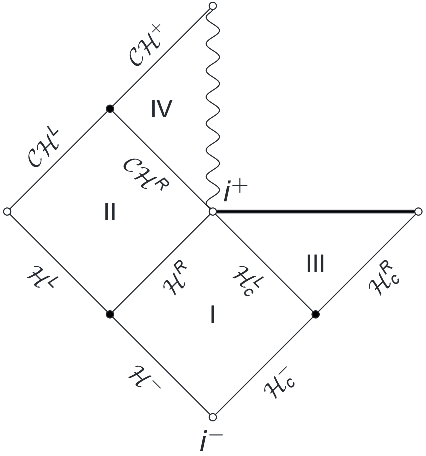
The real scalar field

The charged scalar field

Summary and outlook

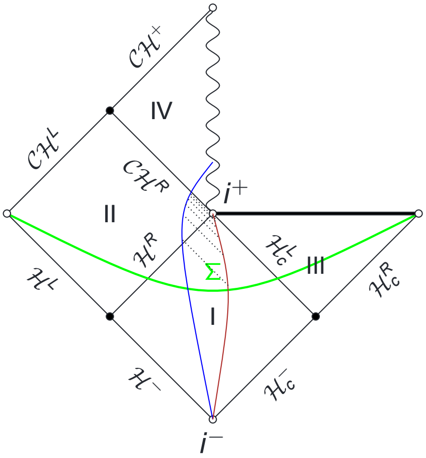
MOTIVATION

The RNdS spacetime



- $G_{\nu\rho} + \Lambda g_{\nu\rho} = 8\pi T_{\nu\rho}$
- $g = -f(r)dt^2 + f^{-1}(r)dr^2 + r^2 d\Omega^2$
- $f(r) = -\frac{\Lambda}{3}r^2 + 1 - \frac{2M}{r} + \frac{Q^2}{r^2}$
- Horizons: $\mathcal{CH} \sim r_-$, $\mathcal{H} \sim r_+$, and $\mathcal{H}_c \sim r_c$

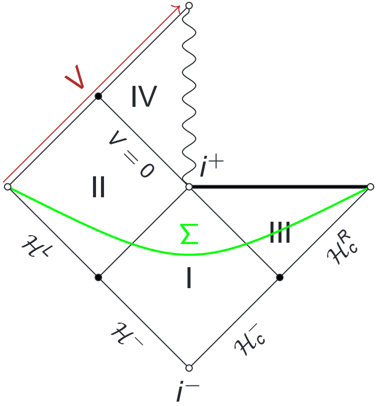
Strong cosmic censorship



- Cauchy horizon beyond which events not determined by initial data on Σ
- Signals reaching $\mathcal{CH}^R$ infinitely blueshifted [Penrose, 1974] $\Leftrightarrow$ Cosmological redshift
- Strong cosmic censorship conjecture (sCC): For generic initial data, metric is inextendible with certain regularity across $\mathcal{CH}^R$

  [Christodoulou, 2008]

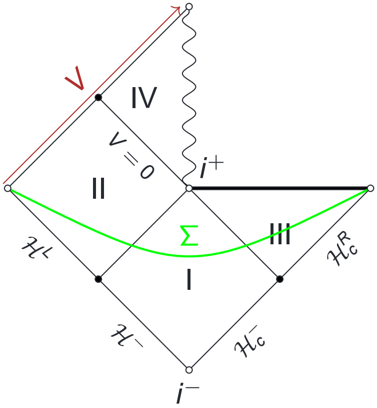
Real scalar perturbations - classically



- Scalar test field: $[\nabla_\nu \nabla^\nu - \mu^2]\phi = 0$
- sCC violated if $\beta = \frac{\alpha}{\kappa_-} > \frac{1}{2}$ [Hintz, Vasy, 2017]
- α spectral gap of quasinormal modes, κ_- surface gravity of $\mathcal{CH}$
- sCC violated classically for large Q

[Cardoso et al., 2017]

Real scalar perturbations - quantum



– Real quantum scalar field in state Ψ
  Hadamard near Σ

$\Rightarrow$ Energy flux $\langle T_{VV} \rangle_\Psi \sim \langle \partial_V \Phi \partial_V \Phi \rangle_\Psi \sim C V^{-2}$

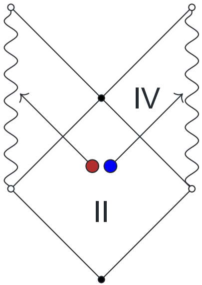
[Hollands et.al., 2020]

– $C \sim \sum_\ell (2\ell + 1) \int_0^\infty \mathrm{d}\omega \, \omega n_\ell(\omega)$

– $n_\ell(\omega)$ function of scattering coefficients

$\Rightarrow$ Is C generically non-zero?

Charged scalar fields



– Charged black hole ⇒ charged scalar field

– Classically: sCC still violated for large Q

[Cardoso et.al., 2018, Dias et.al.,2018]

⇒ Can quantum effects restore sCC?

– Charged field in background electromagnetic field

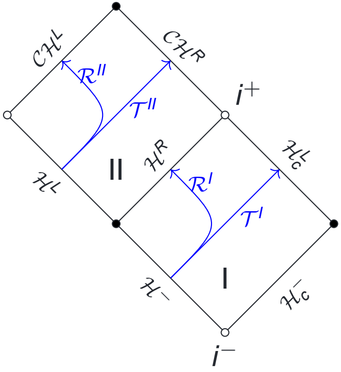
⇒ Non-vanishing charge current

⇒ Intuitive picture: Creation of particle-antiparticle pairs and rapid discharge [Herman, Hiscock, 1994]

⇒ Can quantum effects charge the black hole?

THE REAL SCALAR FIELD

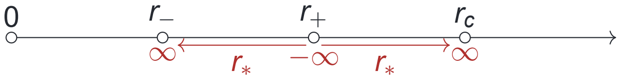
Computation of the scattering coefficients



The "up"-modes

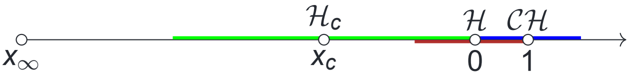
- Klein-Gordon-equation: $\left[\nabla_\nu \nabla^\nu - \mu^2\right] \psi = 0$
- Mode ansatz:
  $\psi_{\omega\ell m} = (4\pi|\omega|)^{-1} r^{-1} Y_{\ell m}(\theta, \phi) e^{-i\omega t} R_{\omega\ell}(r)$
- Equation for $R_{\omega\ell}(r)$:
  $\left[\partial_{r_*}^2 - W(r) + \omega^2\right] R_{\omega\ell}(r) = 0$

$\Rightarrow$ 1d scattering problem

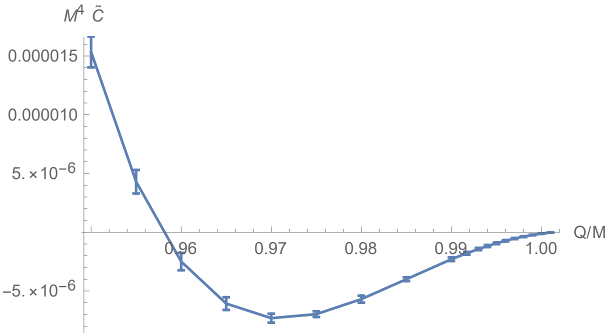


Finding local solutions

- Adapted variable for $\mathcal{H}$: $x = x_\infty \frac{r-r_+}{r-r_o}$ [Suzuki et al.,1999]
- Ansatz: $(r^{-1}R_{\omega\ell})(x) \sim e^{i\omega r_* \frac{x-x_\infty}{1-x_\infty}} h(x)$
- Power-series ansatz: $h(x) = \sum_{n=-\infty}^{\infty} h_n x^n$ with $h_n = 0$ for $n < 0$ and $h_0 = 1$
- Repeat for other horizons



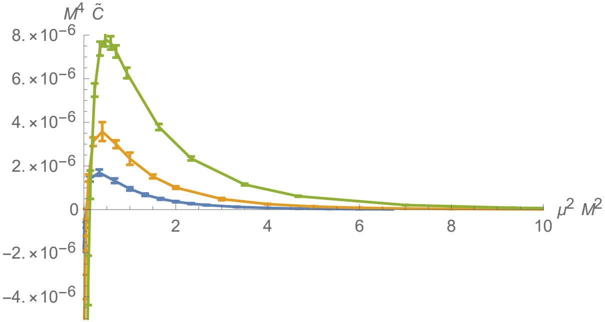
Q/M-dependence of the energy flux



Energy flux $\langle T_{vv}\rangle_{\mathrm{U}}$ at $\mathcal{CH}^R$ $(=\kappa_-^2\, C)$ for the massless real scalar as a function of Q

[Hollands, CK, Zahn, 2020]

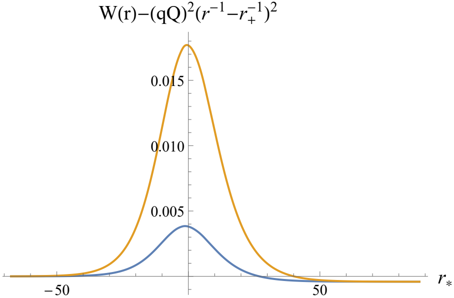
$\mu^2 M^2$-dependence of the energy flux



Energy flux $\langle T_{vv} \rangle_{\mathrm{U}}$ at $\mathcal{CH}^R$ for the real scalar generically non-vanishing [Hollands, CK, Zahn, 2020]

THE CHARGED SCALAR FIELD

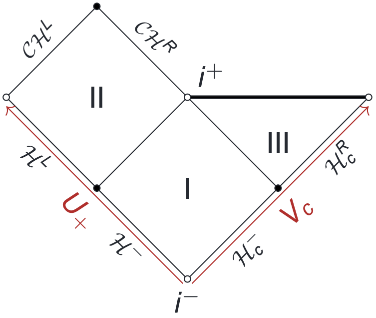
The charged scalar on RNdS



Potential in region I for $\ell = 0$ (blue) and $\ell = 1$ (orange)

- Klein-Gordon-equation: $\left[D_\nu D^\nu - \mu^2\right]\psi = 0$
- $D_\nu = \nabla_\nu - iqA_\nu$, and $A = -\frac{Q}{r}\mathrm{d}t$
- Mode ansatz:
  $\psi_{\omega\ell m} = (4\pi|\omega|)^{-1} r^{-1} Y_{\ell m}(\theta,\phi) e^{-i\omega t} R_{\omega\ell}(r)$
- Equation for $R_{\omega\ell}(r)$:
  $\left[\partial_{r_*}^2 - W(r) + (\omega - qQ/r)^2\right] R_{\omega\ell}(r) = 0$

The current in the Unruh vacuum



- Expansion of quantum field in Unruh modes:

$$\Phi(x) = \sum_J \int_0^\infty \mathrm{d}\omega \left(\psi^U_{\omega J}(x) a_{\omega J} + \psi^U_{(-\omega)J}(x) b^\dagger_{\omega J} \right)$$

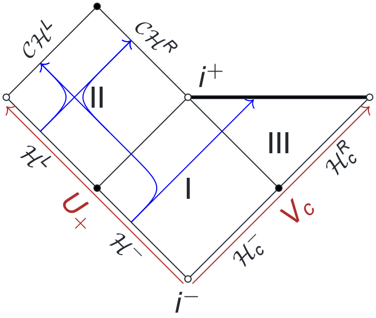
⇒ Unruh vacuum: $a_{\omega J}|0\rangle_\mathrm{U} = b_{\omega J}|0\rangle_\mathrm{U} = 0$

- Charge current:

$$j_\nu(x) = iq(\Phi D^*_\nu \Phi^* - \Phi^* D_\nu \Phi)(x)$$

- Hadamard point-split renormalization

The current in the Unruh vacuum



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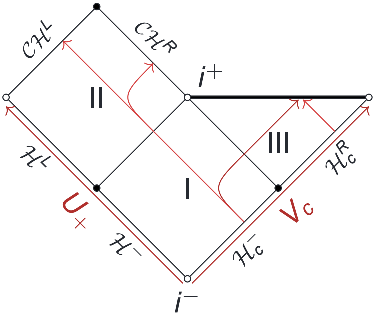

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- Charge current:

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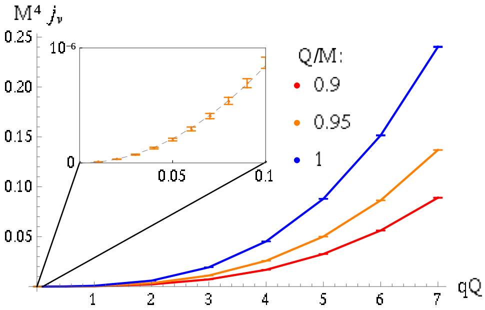
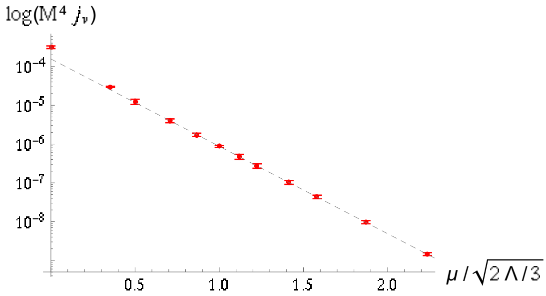
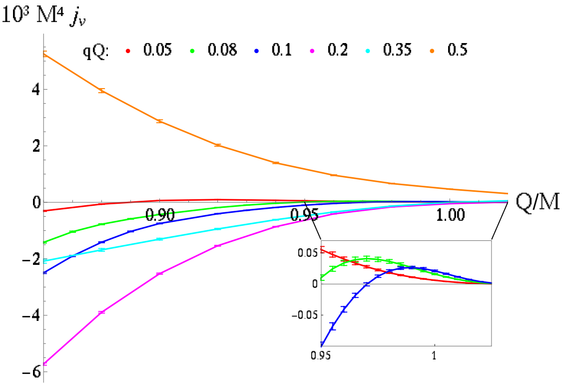
At the event horizon...





- Charge current through $\mathcal{H}^R$ always positive $\Rightarrow$black hole is discharged

...and the Cauchy horizon



– Charge current can be negative

⇒ Charging of black hole

– Charge current positive near maximal Q

⇒ Cannot overcharge black hole

SUMMARY AND OUTLOOK

Summary

- Real scalar field: $C \neq 0$ generically

$\Rightarrow$ sCC restored by quantum effects

- Sign of C depends on scalar field mass and black hole charge

$\Rightarrow$ Final fate of an observer approaching $\mathcal{CH}^R$ parameter-dependent

- Charged scalar:

$\mathcal{H}^R$	$\mathcal{CH}^R$
$\langle j_V \rangle_U$ always positive	$\langle j_V \rangle_U$ can be negative
$\Rightarrow$Black hole discharges	$\Rightarrow$Black hole can be charged

Outlook

RNdS	Kerr-de Sitter
Black hole charge Q	Black hole angular momentum a
spherically symmetric	axisymmetric
static exterior region	stationary exterior region
Charge current $\langle j_V \rangle$	Angular momentum current $\langle T_{V\varphi_-} \rangle$

- Construction of the Unruh state on Kerr-de Sitter [CK: 2022]
- State-independence of the leading divergence of $\langle T_{\mu\nu} \rangle$ at $\mathcal{CH}^R$
- Numerical computation of $\langle T_{V\varphi_-} \rangle_{U-C}$

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THANK YOU FOR YOUR ATTENTION

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