INFRARED FINITE SCATTERING

IN QFT & QUANTUM GRAVITY

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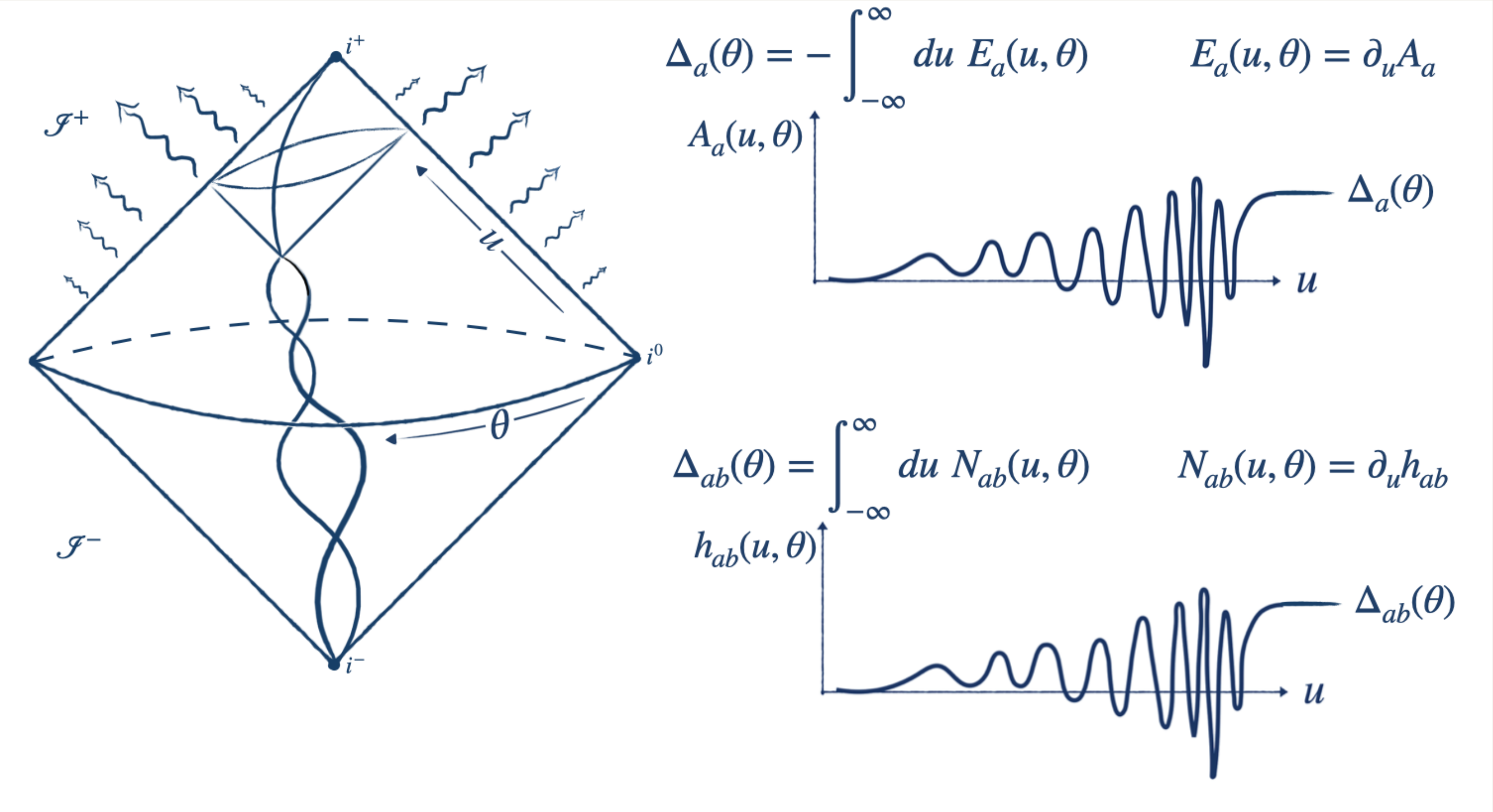
2022

SCATTERING: ASYMPTOTIC FLATNESS

1. generic scattering of particles or fields from past to future infinity

2. The radiation field has a long time tail or a zero frequency part



Massive QED: Asymptotic Fields

Massive charged scalar field φ coupled to electromagnetism

- Massive scalar asymptotics in the limit to $i^{\pm}$ described by $b(p), b^{\dagger}(p), c(p), c^{\dagger}(p)$, where p lies on a unit hyperboloid. *(usual Fourier description)*

- EM vector potential near $\mathscr{I}^{\pm}$:
  $A_a \sim A_A(u,\theta) + O(1/r)$
  Radiative field: $E_A = \partial_u A_A$

- EM memory:
  $\Delta_A(\theta) \sim A_A(u \to +\infty) - A_A(u \to -\infty) = -\int du E_A(u,\theta)$

EM: Fock space

Quantum operator-valued *distribution*: $\mathbf{E}_A(u,\theta)$.

- Vacuum state: $|\omega_0\rangle$ *(Poincaré invariant)*: $\langle\omega_0|\mathbf{E}_{1A}\mathbf{E}_{2B}|\omega_0\rangle \propto \frac{\delta(\theta_1,\theta_2)}{(u_1-u_2-i0^+)^2} q_{AB}$
- one-photon Hilbert space: test "wavefunctions" $s_A(u,\theta)$

$$\|s\|^2 \propto \int d^2\theta \int_0^\infty d\omega\, \omega |s(\omega,\theta)|^2$$

- If wavefunction has memory then $s_A \sim \frac{\Delta_A(\theta)}{\omega}$ and $\|s\|^2$ *diverges logarithmically*.
  No states with memory in standard Fock space!

EM: Memory Hilbert Spaces

Use coherent states with memory

- Classical solution $\mathcal{E}_A(u,\theta)$ with memory $\Delta_A(\theta) = -\int du \mathcal{E}_A(u,\theta)$
- $\mathbf{E}_A \mapsto \mathbf{E}_A + \mathcal{E}_A \mathbf{1}$ (*automorphism of the operator algebra*)
- New Hilbert space of states $\mathscr{F}_\Delta$ with memory $\Delta_A(\theta)$
- $\mathscr{F}_\Delta$ is *unitarily-equivalent* to $\mathscr{F}_{\Delta'}$ if and only if $\Delta_A(\theta) = \Delta'_A(\theta)$
- *Uncountably-many* inequivalent Hilbert spaces!

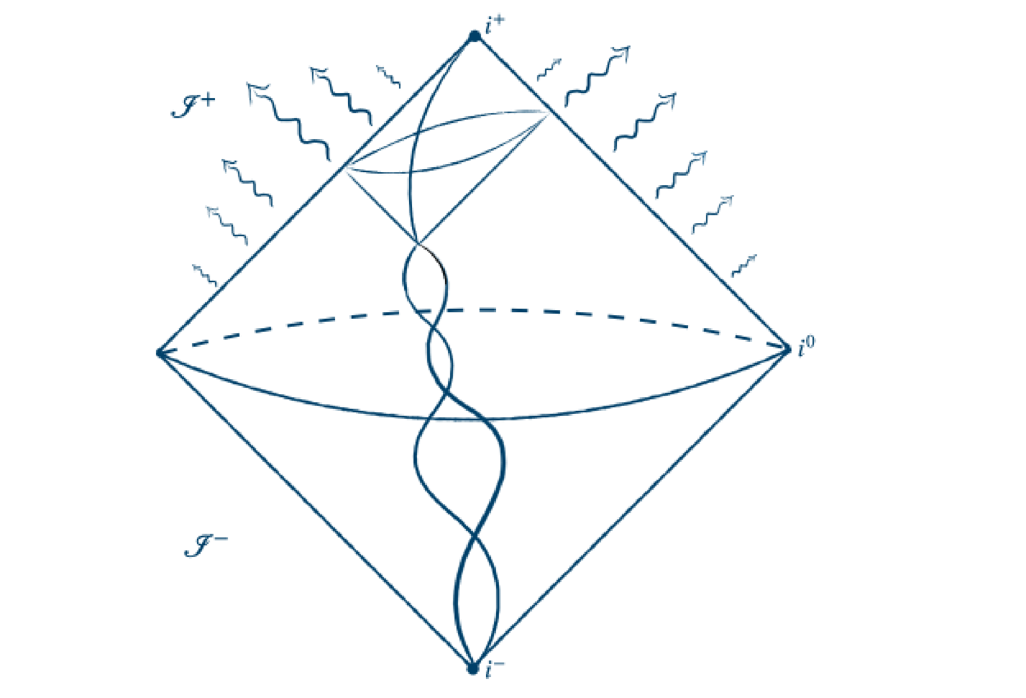
MASSIVE QED: SCATTERING

- EM with no sources: $\Delta_A^{out}(\theta) = \Delta_A^{in}(-\theta)$; antipodal map $\theta \mapsto -\theta$ on $\mathbb{S}^2$
- EM with *classical* source: $\Delta_A^{out}(\theta) = \Delta_A^{in}(-\theta) +$ *classical* current term
- EM with *massive quantum* field: given a $\Delta_A^{in}(\theta)$ *no* definite out-memory!
- If we *pretend* that all states are in the standard Fock space $\mathscr{F}_0$ then S-matrix diverges (*Weinberg soft theorem*)

Idea: build Hilbert spaces using *conserved* quantities, not memory.

CONSERVED CHARGES AT SPATIAL INFINITY

- $\mathcal{Q}_{i^0}(\theta) = \mathcal{Q}_{i^-}(\theta) + \mathscr{D}^A \Delta_A(\theta)$
- $\mathcal{Q}^{out}_{i^0}(\theta) = \mathcal{Q}^{in}_{i^0}(-\theta)$



Find Hilbert spaces of definite charge $\mathcal{Q}_{i^0}(\theta)$

Faddeev-Kulish Hilbert space

1. add *any* EM state $|\psi\rangle \in \mathscr{F}_\Delta$ with memory chosen so that $\mathscr{Q}_{i^0}(\theta) = 0 = \mathscr{Q}_{i^-}(\theta; \{p_i, q_i\}) + \mathscr{D}^A \Delta_A(\theta; \{p_i, q_i\})$

2. Then the *dressed* state $\int d^3p_1 \dots d^3q_n w(\{p_i, q_i\}) |p_1, \dots, p_n, q_1, \dots, q_n\rangle \otimes |\psi\rangle_{\Delta(\theta; \{p_i, q_i\})}$ has zero charge at spatial infinity.

3. Faddeev-Kulish Hilbert space $\mathscr{H}^{FK}$ by direct summing over particle-antiparticle number.

Charge conservation ensures that this Hilbert space scatters to itself! So the S-matrix $S : \mathscr{H}^{FK} \to \mathscr{H}^{FK}$ is well-defined.

FK Hilbert spaces: issues

- Total charge cannot be dressed away; memory is $\ell \geq 1$; always need equal number of particles and antiparticles *(hide particles behind the moon)*

- Can define FK Hilbert spaces with any value of $\mathcal{Q}_{i^0}(\theta)$ but not Lorentz-invariant; angular momentum/spin of states is undefined! *("Lorentz is spontaneously broken")*

- Massive fields always come with a radiative photon cloud; can hide the cloud at very low frequencies but always have to send it in.

Massless QED & Linearized Gravity

- Massless QED: solve $\mathcal{Q}_{i^0}(\theta) = 0 = \mathcal{J}(\theta; \{p_i, q_i\}) + \mathscr{D}^A \Delta_A(\theta; \{p_i, q_i\})$
- $\mathcal{J}(\theta; p) = \delta(\theta, \hat{p})$ implies $\Delta_A \sim 1/(\theta - \hat{p})$ *(collinear divergence)*; Δ_A is not L^2; *photon dressing has infinite energy!*
- Linearized gravity: solve $\mathcal{Q}_{i^0}(\theta)\big|_{\ell \geq 2} = 0 = \mathcal{T}(\theta; \{p_i, q_i\}) + \mathscr{D}^A \mathscr{D}^B \Delta_{AB}(\theta; \{p_i, q_i\})$
- $\mathcal{T}(\theta; p) = \delta(\theta, \hat{p})$ implies $\Delta_{AB} \sim \log(\theta - \hat{p})$ *(collinear divergence)*; Δ_{AB} is L^2; *dressing has finite energy!*
- $\mathcal{Q}_{i^0}(\theta)\big|_{\ell \geq 2} = 0$ is not Lorentz-invariant, so no angular momentum/spin for these states

Nonlinear GR

Radiative field is News N_{AB}; gravitational memory is $\Delta_{AB}(\theta) = \int du N_{AB}(u, \theta)$

$$\mathcal{Q}_{i^0}(\theta) = \int du \, N_{AB}(u, \theta) N^{AB}(u, \theta) + \mathscr{D}^A \mathscr{D}^B \Delta_{AB}(\theta)$$

- The memory Hilbert spaces $\mathscr{F}_\Delta$ still exist and memory is not conserved
- *gravity sources itself*; any kind of dressing will further contribute to the charge

Theorem: The only *Hadamard* state of fixed $\mathcal{Q}_{i^0}(\theta)$ is the *vacuum* with $\mathcal{Q}_{i^0}(\theta) = 0$.

So no non-vacuum state is in *any* FK Hilbert space!

ALGEBRAIC SCATTERING

- In/out states defined as lists of correlation functions $\omega(\mathbf{O}_1, \ldots, \mathbf{O}_n)$
- Any such state lives in some Hilbert space, but *not all states* can be put in a *single Hilbert space*
- Evolution/scattering is given by a $\$$-map: $\$: \omega^{in} \mapsto \omega^{out}$
- conservation of probability: $\omega^{in}(\mathbf{1}) = 1$ implies $\omega^{out}(\mathbf{1}) = 1$;
  no information loss: $\$$ maps pure in-states to pure out-states

How to *practically* compute such a $\$$? *¯_(ツ)_/¯* Usual methods like Feynman diagrams/LSZ reduction do not work; Hawking (1976) has interesting ideas, but still assumes a single Hilbert space

don't take a vacation yet! Or maybe take a vacation and think about it!!