Black Holes Decohere Quantum Superpositions

Daine L. Danielson, Gautam Satishchandran, Robert M. Wald
The University of Chicago

arXiv:2205.06279 [hep-th] to appear in IJMPD (2022)
and forthcoming generalizations (2022).

GR23 Beijing, July 2022. daine@uchicago.edu

Introduction

- Black holes are well known to be destroyers of quantum coherence.
- The purpose of this talk is to show them to be even more insidious destroyers of quantum coherence than has previously been known.
- We will find that a quantum spatial superposition of a massive, or electromagnetically charged, body must suffer a rate of decoherence due to the mere presence of a black hole in its vicinity!

Introduction

- In the Minkowski vacuum, it is possible to maintain the quantum coherence of a wavefunction occupying a superposition of positions, by minimizing the entangling radiation emitted by the superposed matter.
- We describe a new phenomenon whereby, outside a black hole, any quantum spatial superposition of matter will suffer a constant rate of decoherence. This decoherence is unavoidable, and will persist even if the energy radiated by the superposition is absolutely minimized.

Outline

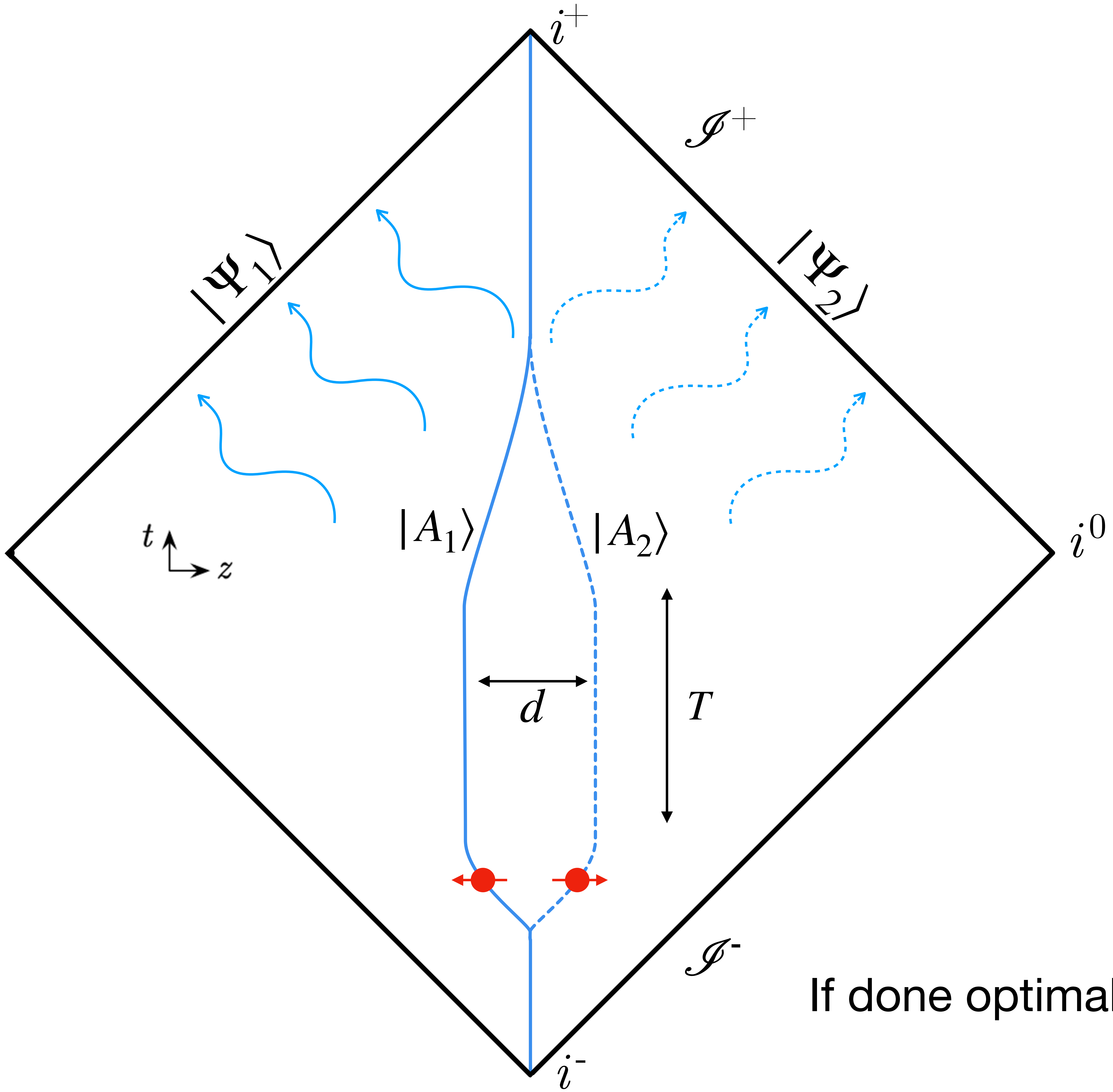
- Coherent spatial superpositions in Minkowski spacetime (E&M)
  - Gedankenexperiment
  - How to avoid decoherence due to radiation
- Decoherence in a black hole spacetime
  - Maxwell's equations on the event horizon
  - Field of a displaced point charge
  - Quantization of the field on the event horizon
  - Black hole decoherence effect
    - Maxwell case
    - Quantum gravitational case (linearized)

Flat Spacetime

- To understand this effect, let's first consider what must be done to maintain the quantum coherence of a spatial superposition of an electromagnetically charged particle in Minkowski spacetime.
- We consider a gedankenexperiment similar to that discussed in the preceding talk by Gautam Satishchandran, and in Phys. Rev. D 105, 086001 ("Gravitationally Mediated Entanglement: Newtonian Field vs. Gravitons").
  - Our gedankenexperiment will be simpler: there will be only "Alice," and no "Bob."

Final state: $\frac{1}{\sqrt{2}}\left(|\uparrow, A_1\rangle_{i^+} \otimes |\Psi_1\rangle_{\mathscr{I}^+} + |\downarrow, A_2\rangle_{i^+} \otimes |\Psi_2\rangle_{\mathscr{I}^+}\right)$

Decoherence: $\mathscr{D} = 1 - |\langle \Psi_1 | \Psi_2 \rangle_{\mathscr{I}^+}|$



If done optimally, i.e., minimal decoherence

$\Longrightarrow \mathscr{D} \sim 0$

Maintaining coherence in flat spacetime

- To avoid decoherence, Alice must avoid radiating "which-path" information into the electromagnetic field. Her particle carries a charge q, and the superposition is of size d.
- To maintain coherence, Alice must ensure the number of photons difference between the outgoing field states is $\langle N \rangle \ll 1$. If Alice recombines sufficiently gradually, $T \gg qd/\sqrt{\epsilon_0 c^3 \hbar}$, she can avoid radiating such "entangling photons."
- In Minkowski spacetime Alice can coherently recombine her wave function, by performing her experiment so gradually that Alice's particle radiates almost nothing from either path, and $|\Psi_1\rangle_{\mathscr{I}^+} \sim |\Psi_2\rangle_{\mathscr{I}^+} \sim |0\rangle_{\mathscr{I}^+}$. We will see that this will become impossible in the presence of a black hole.

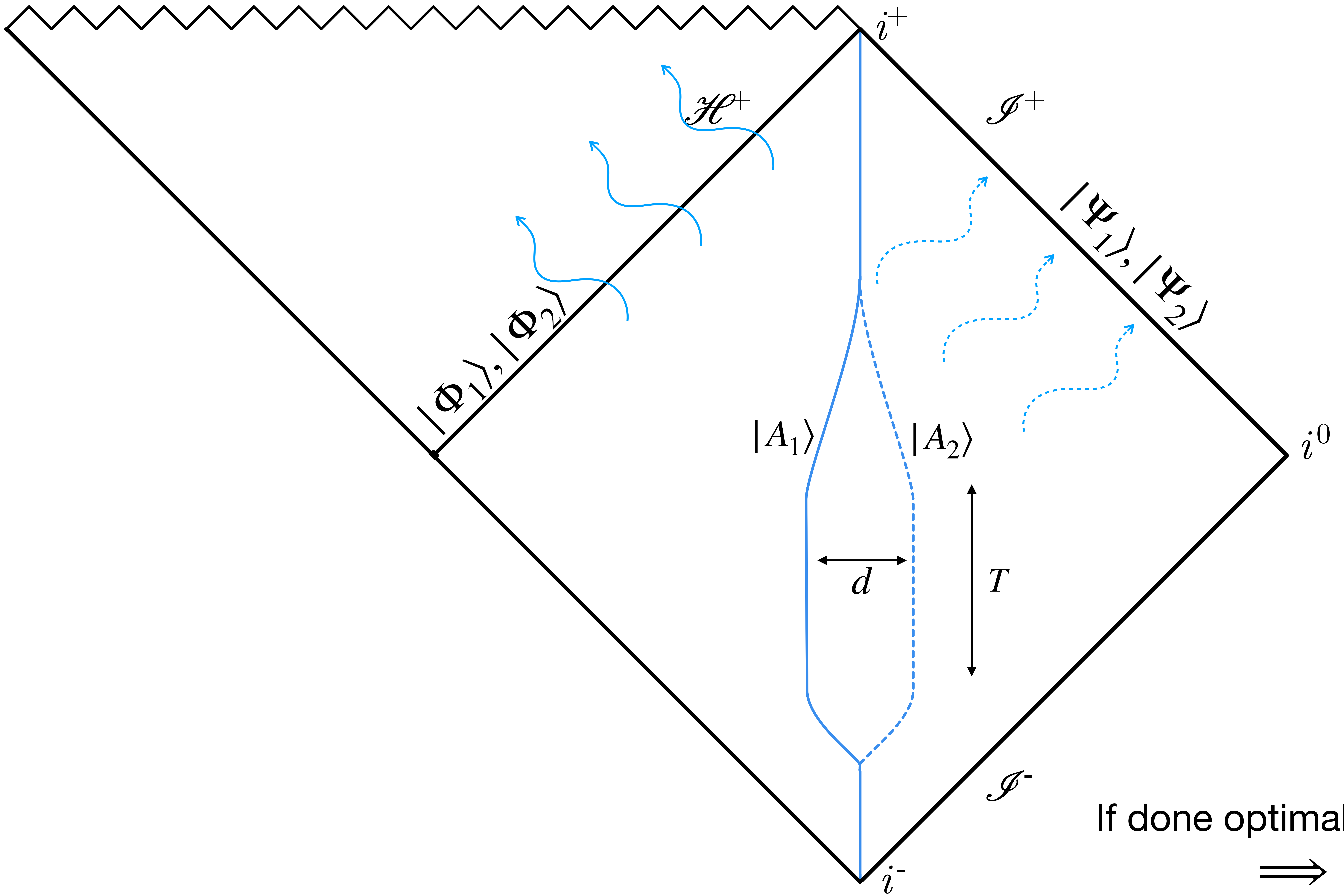
Repeat the gedankenexperiment outside a black hole.

Gedankenexperiment outside a black hole

- We now repeat the gedankenexperiment in the case where there is a black hole in the vicinity of Alice.
- To prevent Alice and her experiment from falling into the black hole, equip her with rockets so that her laboratory is kept stationary at a fixed radius outside the event horizon.
- Again, Alice creates a spatial superposition and maintains it for a time T, and then carefully recombines it to look for signs of decoherence.
- The analysis proceeds largely as before, with one crucial difference: now, radiation can propagate through the event horizon as well as to null infinity.

Final state: $\frac{1}{\sqrt{2}}\left(|\uparrow, A_1\rangle_{i^+} \otimes |\Psi_1\rangle_{\mathscr{I}^+}|\Phi_1\rangle_{\mathscr{H}^+} + |\downarrow, A_2\rangle_{i^+} \otimes |\Psi_2\rangle_{\mathscr{I}^+}|\Phi_2\rangle_{\mathscr{H}^+}\right)$

Decoherence: $\mathscr{D}_{\mathrm{BH}} = 1 - |\langle\Psi_1|\Psi_2\rangle_{\mathscr{I}^+}\langle\Phi_1|\Phi_2\rangle_{\mathscr{H}^+}|$



If done optimally, i.e., minimal decoherence

$$\Longrightarrow \mathscr{D}_{\mathrm{BH}} \sim 1 - |\langle\Phi_1|\Phi_2\rangle_{\mathscr{H}^+}|$$

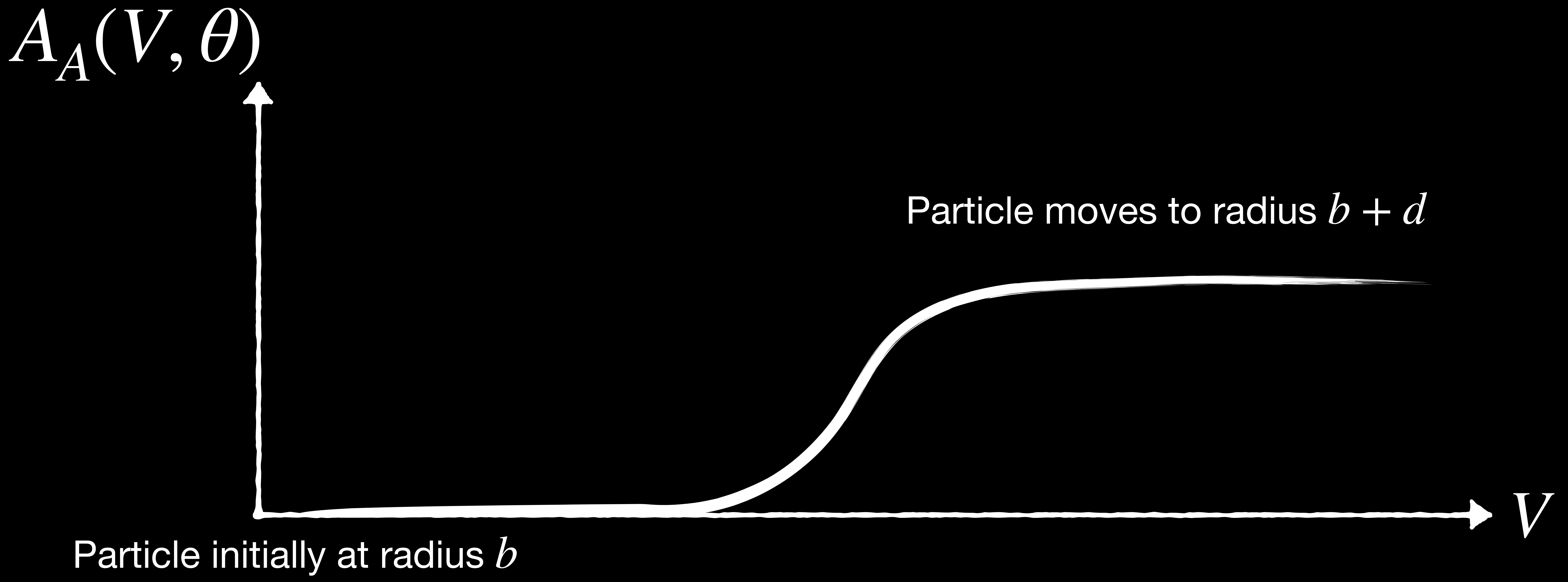
Schwarzchild geometry

- Let $q_{ab}(b)$ be the metric on the 2-sphere centered on Schwarzchild radial coordinate b.
- Consider the Kruskal-Szekeres metric, $g_{ab} = -2f(b)(dU)_{(a}(dV)_{b)} + q_{ab}$, where $f(b) = \dfrac{4r_s^3 e^{-b/r_s}}{b}$.
- Affine time V gives a vector field $(\partial_V)^a$ tangent to the horizon, and U gives a “radial” null vector field $(\partial_r)^a \equiv -(\partial_U)^a/f(b)$ penetrating the horizon. Together these satisfy $(\partial_r)_a(\partial_V)^a = 1$.
- We denote angular indices on the 2-sphere with capital roman letters: $A, B, C, \ldots$
- D_A is the covariant derivative with respect to the 2-sphere cross-sections of the horizon.

Maxwell's equation on the event horizon

- E_A is the pullback of the electric field to the event horizon. A change in E_A indicates radiation entering the black hole.
- A Coulomb field gives rise to E_r on the horizon.
- On the horizon, E_A (radiation into the black hole) and E_r (the Coulomb field) are related by Maxwell's equations: $D^A E_A = -\partial_V E_r$.
- The radial electric field at a distance b is, to leading order on the horizon, $E_r \sim \dfrac{q}{b^2}$.

  If that charge becomes displaced by a distance d, $\Delta E_r \sim \dfrac{qd}{b^3}$. Thus, $\Delta A_A \sim r_s^2 \dfrac{qd}{b^3}$.



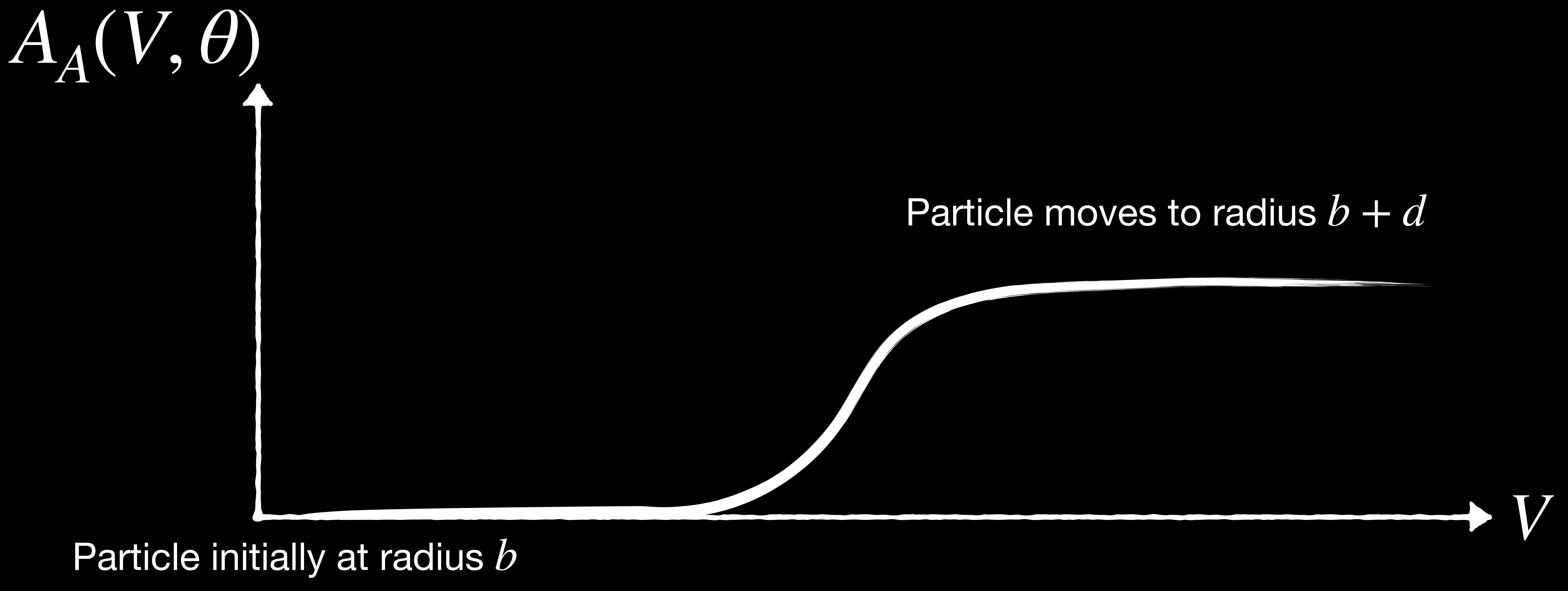
$$D^A E_A = -\partial_V E_r \qquad E_A = \partial_V A_A$$

$$\Delta E_r \neq 0 \implies \Delta A_A \neq 0$$

Quantization on the event horizon

- Let's consider the implications for the quantum electromagnetic field.
- We should quantize with respect to the Unruh vacuum, this being the vacuum of a Schwarzchild black hole formed by gravitational collapse. However, we will be concerned only with low frequency phenomena $(\omega \ll 1)$, in which case the Unruh and Hartle-Hawking vacua are essentially indistinguishable.
- In the Hartle-Hawking Fock space we have the inner product on the horizon,

$$\langle A_{1,B} | A_{2,C} \rangle_{\mathscr{H}^+} \equiv \frac{2c}{\hbar} \int_{S^2} r_{\rm s}^2 d\Omega \int_0^\infty \frac{\omega d\omega}{2\pi} \; q^{BC} \overline{\hat{A}_{1,B}(\omega, x^A)} \hat{A}_{2,C}(\omega, x^A)$$



$$D^A E_A = -\partial_V E_r \qquad E_A = \partial_V A_A$$

$$\Delta E_r \neq 0 \implies \Delta A_A \neq 0 \implies \hat{A}_A(\omega, \theta) \sim \frac{1}{\omega}$$

$$\langle N \rangle = \|A\|^2_{\mathscr{H}^+} = \infty$$

- Energy content can be made absolutely negligible… but not photon number!
- This is reminiscent of the memory effect, as discussed in Kartik Prabhu's talk.

Soft horizon photons

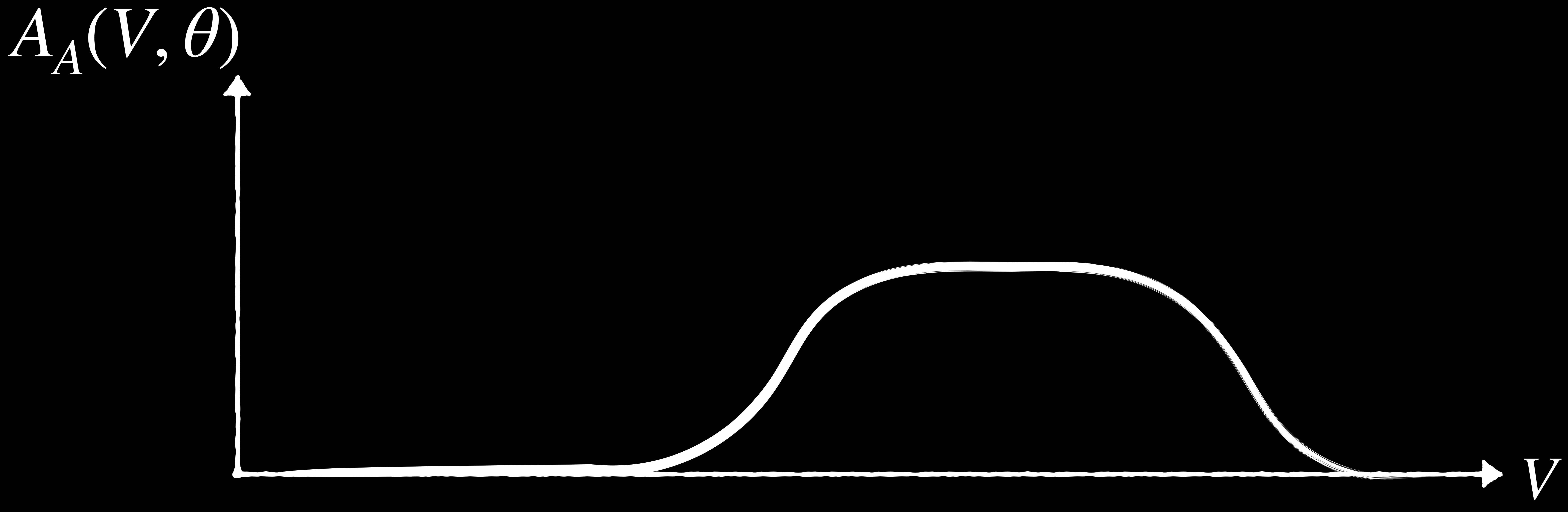
- A charge-current positioned a distance b outside the black hole sources a retarded solution A_A on the horizon. This will contain an expected number of "horizon photons" $\langle N \rangle = ||A_A||^2_{\mathscr{H}^+}$.

- Now suppose Alice has a classical charge initially at some fixed radius b from the black hole, and quasi statically displaces it to another radius $b + d$. The resulting field configuration on the horizon will not return to its initial at late times, due to the shift in the Coulomb field.

- This implies the Fourier transform $\hat{A}_A$ will diverge like $1/\omega$ as $\omega \to 0$, resulting in $||A_A||^2_{\mathscr{H}^+} = \infty$, an infinite number of soft horizon photons. This is reminiscent of the memory effect, as discussed in Kartik Prabhu's talk.

Alice's moving particle outside a black hole

- The case of relevance to Alice is one in which she displaces her from its initial radius b to a new radius $b + d$ by a time $t = 0$, and then after a time $t = T$ returns it quasi-statically to its initial position. We consider $d \ll b$.
- Then there is no infrared divergence, but $\langle N \rangle$ is nonetheless very large when T is very large.



$$D^A E_A = -\partial_V E_r \qquad E_A = \partial_V A_A$$

$$\Delta E_r \neq 0 \implies \Delta A_A \neq 0 \implies \hat{A}_A(\omega,\theta) \sim \frac{1}{\omega}$$

$$\langle N \rangle = \|A_A\|^2_{\mathscr{H}^+} \sim \ln \Delta V$$

Alice's moving particle outside a black hole

- The motion of Alice's particle must produce a total number of soft horizon photons $\langle N \rangle = ||A_A||^2_{\mathscr{H}^+} \sim \dfrac{G^4 M^4 q^2 d^2}{\hbar c^9 b^6} \ln V$.
- In terms of Killing time v, $V = e^{\kappa v}$ where κ is the surface gravity. We can imagine Alice to be sufficiently far away that the redshift factor is approximately 1, so that Killing time is very nearly Alice's proper time.
- Alice's experiment necessitates the production of $\langle N \rangle \sim \dfrac{G^3 M^3 q^2 d^2}{\hbar c^5 b^6} T$ soft horizon photons.

Black hole decoherence effect

for a charged particle

- Now we are ready to reanalyze the gedankenexperiment, outside of a black hole.
- One branch of Alice's wave function remains at rest in her laboratory to produce an electromagnetic state on the horizon $|\Phi_1\rangle_{\mathscr{H}^+} = |0\rangle_{\mathscr{H}^+}$.
- Alice quasi-statically separates the other branch to a distance d, then reverses that separation after a proper time T, to produce a horizon state $|\Phi_2\rangle_{\mathscr{H}^+}$ with expected photon number $\langle N \rangle \sim \dfrac{G^3 M^3 q^2 d^2}{\hbar c^5 b^6} T$.
- If $\langle N \rangle \gtrsim 1$ then $|\Phi_1\rangle_{\mathscr{H}^+}$ is nearly orthogonal to $|\Phi_2\rangle_{\mathscr{H}^+}$ and Alice's particle will be decohered.

Black hole decoherence effect

for a charged particle

- Therefore Alice's superposition will decohere after a time
$$T_D \sim \frac{\hbar c^6 b^6}{G^3 M^3 q^2 d^2} \sim 10^{43}\ \text{years} \left(\frac{b}{\text{a.u.}}\right)^6 \cdot \left(\frac{M_\odot}{M}\right)^3 \cdot \left(\frac{e}{q}\right)^2 \cdot \left(\frac{\text{m}}{d}\right)^2 .$$
- Thus, if our Sun were a black hole and if one separated an electron into two components one meter apart in a laboratory experiment on Earth, it would not be possible to maintain the coherence of the electron for more than 10^{43} years. On the other hand, if this experiment were done at $b = 6GM/c^2$, then $T_D \sim 5\,\text{minutes}$.

Black hole decoherence effect
for a massive particle

- A similar effect holds for any *massive* body, due to the (linearized) quantum gravitational field sourced by a spatially superposed mass.
- Replace the time-varying "electric field" on the horizon with the perturbation of the electric Weyl tensor $E_{ab} = C_{aVbV}$. Bianchi identity gives $D^A E_{AB} = -\partial_V E_{rB}$, $D^A E_{rA} = -\partial_V E_{rr}$, and $D^A D^B E_{AB} = \partial^2 E_{rr}$. The role of the potential is now played by h_{AB}, with $E_{AB} = -\frac{1}{2}\partial_V^2 h_{AB}$.

Black hole decoherence effect

for a massive particle

- The analysis proceeds exactly as before. A superposed mass will be decohered by soft horizon gravitons in a time

$$T_D^{\mathrm{GR}} \sim \frac{\hbar c^{10} b^{10}}{G^6 M^5 m^2 d^4} \sim 10\ \mu\mathrm{s} \left(\frac{\mathrm{b}}{\mathrm{a.u.}}\right)^{10} \cdot \left(\frac{\mathrm{M}_{\odot}}{\mathrm{M}}\right)^{5} \cdot \left(\frac{\mathrm{M}_{\mathrm{Earth}}}{\mathrm{m}}\right)^{2} \cdot \left(\frac{\mathrm{R}_{\mathrm{Earth}}}{\mathrm{d}}\right)^{4}.$$

- Thus, if the Sun were a black hole and the Earth occupied a quantum state with its center of mass spatially superposed by a separation on the order of its own radius, this superposition would decohere due to the presence of the black hole in about 10 microseconds. Of course, it would not be easy to put the Earth into such a quantum superposition.

Black holes harvest information about quantum spatial superpositions outside their horizons, by means of the long-range fields sourced by the superposed matter.

This exerts a fundamental rate of decoherence these superpositions. Eventually, a black hole will decohere any quantum superposition.