The Weyl double copy in maximally symmetric spacetimes

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Thank A. Luna, R. Monteiro, N. Obers and X. Qian for useful discussions/comments

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What is the Weyl double copy?

- BCJ duality [*Bern, Carrasco and Johansson,2008*]
- The Weyl double copy [*Luna,Monteiro,Nicholson and O'Connell,2019*]

$$\Psi_{ABCD} = \frac{\Phi_{(AB}\Phi_{CD)}}{S} \tag{1}$$

- Kerr-Schild DC in (A)dS (T, $\not{T}$) [*Carrillo-Gonzalez et al.,2018*]
- D $\rightarrow$ N [*Godazgar et al.,2021*]

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- What about the Weyl double copy in (A)dS background?

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- Is it possible to decode the information of gravitational waves (vacuum N) into single and zeroth copies?

Massless free-fields in spinor formalism

- Spin-$n/2$ massless free-field equations

$$\nabla^{A_1 A_1'} \mathcal{S}_{A_1 A_2 \ldots A_n} = 0 \tag{2}$$

- Weyl fields ($n = 4$, **s**=2)

$$C_{abcd} = C_{AA'BB'CC'DD'} = \Psi_{ABCD}\varepsilon_{A'B'}\varepsilon_{C'D'} + \bar{\Psi}_{A'B'C'D'}\varepsilon_{AB}\varepsilon_{CD} \tag{3}$$

  - general form

$$\begin{aligned}\Psi_{ABCD} = {} & \psi_0 \iota_A \iota_B \iota_C \iota_D - 4\psi_1 o_{(A}\iota_B \iota_C \iota_{D)} + 6\psi_2 o_{(A} o_B \iota_C \iota_{D)} \\ & - 4\psi_3 o_{(A} o_B o_C \iota_{D)} + \psi_4 o_A o_B o_C o_D .\end{aligned} \tag{4}$$

- Maxwell fields ($n = 2$, **s**=1)

$$F_{ab} = F_{AA'BB'} = \Phi_{AB}\varepsilon_{A'B'} + \bar{\Phi}_{A'B'}\varepsilon_{AB} \tag{5}$$

- Dirac-Weyl(DW) fields ($n = 1$, **s**=1/2)

$$P_{ab} = \xi_A \xi_B \varepsilon_{A'B'} \tag{6}$$

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- Introduce a general map

$$\Psi_{ABCD} = \frac{\xi_{(A}\eta_B\zeta_C\chi_{D)}}{S_{14}} \tag{7}$$

The Weyl double copy in maximally symmetric spacetimes

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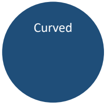
- Other higher spin than $1/2$ massless free-fields

$$e.g. \qquad \Phi_{AB} = \frac{\xi_{(A}\eta_{B)}}{S_{12}} \tag{8}$$

The case for vacuum type N solutions

$$\nabla^{AA'}\Psi_{ABCD} = 0$$

$$\nabla^{AA'}\xi_{A} = 0$$



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Curved

$$\Psi_{ABCD} = \frac{\xi_A\xi_B\xi_C\xi_D}{S_{14}}$$

$$\psi_4 o_A o_B o_C o_D = \frac{\xi^4}{(S_{12})^3} o_A o_B o_C o_D$$

$S_{12} = S_{24} = (S_{14})^{1/3}$
arXiv:2204.01907

$\Downarrow$

ξ , S_{12} identified

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$\xi_{\mathrm{A}} = \xi o_{\mathrm{A}}$

$$\Phi_{AB} = \frac{\xi_A \xi_B}{S_{12}} = \frac{\xi^2}{S_{12}} o_A o_B, \text{ identified}$$

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⇓

F_{ab}, S_{24}

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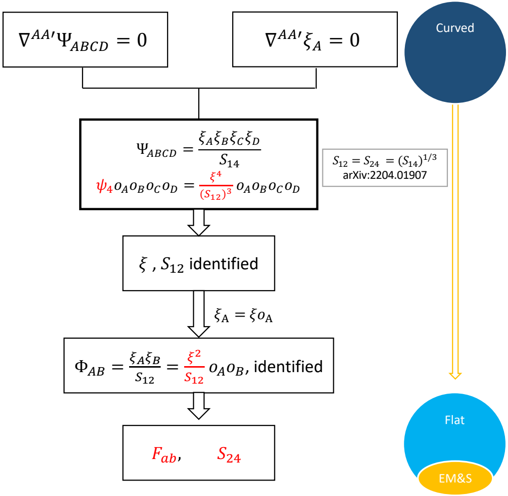

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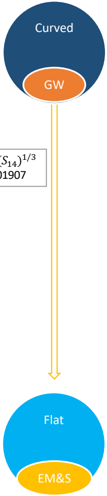


The KN(Λ) class of gravitational waves

- The metric [*Ozsvath et al., 1985*][*Bicak et al.,1999*]

$$\mathrm{d}s^2 = -F\mathrm{d}u^2 + 2\frac{q^2}{p^2}\mathrm{d}u\mathrm{d}v - 2\frac{1}{p^2}\mathrm{d}z\mathrm{d}\bar{z} \tag{10}$$

where

$$\begin{aligned} p &= 1 + \frac{\Lambda}{6}z\bar{z}, \qquad q = \left(1 - \frac{\Lambda}{6}z\bar{z}\right)\alpha + \bar{\beta}z + \beta\bar{z}, \\ F &= \kappa\frac{q^2}{p^2}v^2 - \frac{\left(q^2\right)_{,u}}{p^2}v - \frac{q}{p}H, \qquad \kappa = \frac{\Lambda}{3}\alpha^2 + 2\beta\bar{\beta}, \\ H &= H(u, z, \bar{z}) = \left(f_{,z} + \bar{f}_{,\bar{z}}\right) - \frac{\Lambda}{3p}(\bar{z}f + \xi\bar{f}). \end{aligned} \tag{11}$$

The KN(Λ) class of gravitational waves

- The results

$$\Psi_4 = \frac{1}{72}(\Lambda z\bar{z}+6)\left((\Lambda z\bar{z}-6)\alpha - 6(\bar{z}\beta + z\bar{\beta})\right)\partial_{\bar{z}}^3 \bar{f}$$
$$\phi_2 = \frac{(6+\Lambda z\bar{z})}{6\sqrt{2}}\sqrt{\mathcal{C}(u,\bar{z})\partial_{\bar{z}}^3 \bar{f}(u,\bar{z})} \tag{11}$$
$$S_{24} = S_{12} = \mathcal{C}(u,\bar{z})\frac{\Lambda z\bar{z}+6}{(\Lambda z\bar{z}-6)\,\alpha - 6\left(\bar{z}\beta + z\bar{\beta}\right)}$$

- **NB:**

$$F_{ab} = 2\phi_2 \ell_{[a} m_{b]} + 2\bar{\phi}_2 \ell_{[a}\bar{m}_{b]}. \tag{12}$$

$$2\ell_{[a}m_{b]} = \begin{pmatrix} 0 & 0 & 0 & I \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -I & 0 & 0 & 0 \end{pmatrix}, \quad 2\ell_{[a}\bar{m}_{b]} = \begin{pmatrix} 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 \\ -I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{13}$$

where $I = \frac{6}{6+\Lambda z\bar{z}}$

The RTN(Λ) class of gravitational waves

- The metric [*Diaz et al., 1981*][*Bicak et al., 1999; Griffiths et al., 2002*]

$$
\begin{aligned}
\mathrm{d}s^2 = & -2(A\bar{A} + \psi B)\mathrm{d}u^2 - 2\psi \mathrm{d}u\mathrm{d}v \\
& - 2v\bar{A}\mathrm{d}u\mathrm{d}z - 2vA\mathrm{d}u\mathrm{d}\bar{z} - 2v^2\mathrm{d}z\mathrm{d}\bar{z}, \\
A = & \epsilon z - vf, \quad B = -\epsilon + \frac{v}{2}(f_{,z} + \bar{f}_{,\bar{z}}) + \frac{\Lambda}{6}v^2\psi, \quad \psi = 1 + \epsilon z\bar{z},
\end{aligned}
\tag{14}
$$

where $\epsilon = +1, 0, -1$ correspond to the source of the transverse gravitational waves is time-like, null and space-like respectively, at least in week field limit.

The RTN(Λ) class of gravitational waves

- The results

$$\Psi_4 = \frac{(1+\epsilon z\bar{z})\partial_{\bar{z}}^3 \bar{f}}{2v}$$
$$\phi_2 = \sqrt{\frac{C(u,\bar{z})\partial_{\bar{z}}^3 \bar{f}(u,\bar{z})}{2}\frac{1}{v}}$$
$$S_{24} = S_{12} = \frac{C(u,\bar{z})}{v(1+\epsilon z\bar{z})} \tag{15}$$

- **NB:**

$$F_{ab} = 2\phi_2 \ell_{[a} m_{b]} + 2\bar{\phi}_2 \ell_{[a} \bar{m}_{b]} \tag{16}$$

$$2\ell_{[a} m_{b]} = \begin{pmatrix} 0 & 0 & 0 & v \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -v & 0 & 0 & 0 \end{pmatrix} \qquad 2\ell_{[a} \bar{m}_{b]} = \begin{pmatrix} 0 & 0 & v & 0 \\ 0 & 0 & 0 & 0 \\ -v & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{17}$$

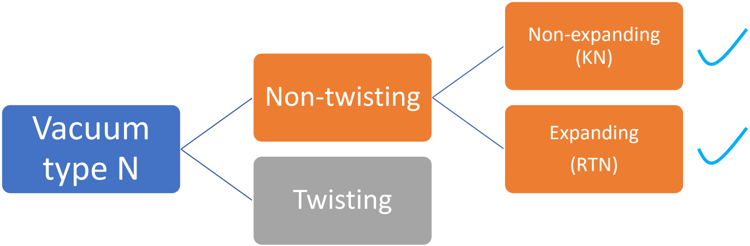
The case for non-twisting vacuum type N solutions

- Conformally invariant field equations in **conformally flat spacetimes**

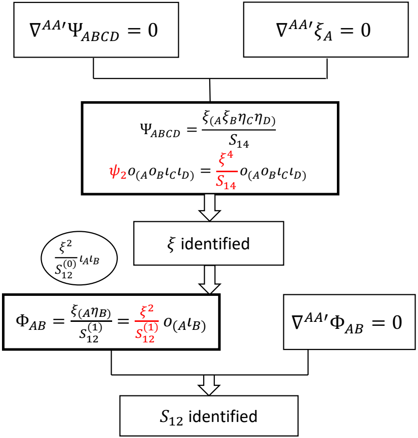
$$\textit{Single copy:} \quad \tilde{\nabla}_a F^{ab} = 0 \tag{18}$$

$$\textit{Zeroth copy:} \quad \tilde{\nabla}^a \tilde{\nabla}_a S_{24} - \frac{1}{6}\tilde{R} S_{24} = 0 \tag{19}$$

Vacuum type N solutions



The case for vacuum type D solutions



$\xi_A = \xi o_A; \eta_A = \xi \iota_A$

The case for vacuum type D solutions

- The general vacuum type D solutions [*Plebanski,Demianski, 1976*]

$$ds^2 = \frac{1}{(p+q)^2}\left(-\frac{1+(pq)^2}{\mathcal{P}}\mathrm{d}p^2 - \frac{\mathcal{P}}{1+(pq)^2}(\mathrm{d}\sigma + q^2\mathrm{d}\tau)^2 - \frac{1+(pq)^2}{\mathcal{L}}\mathrm{d}q^2 + \frac{\mathcal{L}}{1+(pq)^2}(-p^2\mathrm{d}\sigma + \mathrm{d}\tau)^2\right) \quad (20)$$

where the structure functions read

$$\begin{aligned} \mathcal{P} &= (-\frac{\Lambda}{6}+\gamma) + 2np - \epsilon p^2 + 2mp^3 + (-\frac{\Lambda}{6}-\gamma)p^4 \\ \mathcal{L} &= (-\frac{\Lambda}{6}-\gamma) + 2nq + \epsilon q^2 + 2mq^3 + (-\frac{\Lambda}{6}+\gamma)q^4 \end{aligned} \quad (21)$$

The case for vacuum type D solutions

- The results

$$\Psi_2 = 6\psi_2 = -6(m+in)\left(\frac{p+q}{1-ipq}\right)^3$$
$$\phi_1 = \sqrt{\frac{-6C_1(m+in)}{C_2}}\frac{(p+q)^2}{(1-ipq)^2} \sim \left(\frac{p+q}{1-ipq}\right)^2 \tag{21}$$
$$S_{24}^{(1,1)} \sim S_{12}^{(0)} \sim S_{12}^{(2)} \sim \frac{p+q}{1-ipq}$$

- **NB:**

$$F_{ab} = 2\phi_1\left(\ell_{[a}n_{b]} + \bar{m}_{[a}m_{b]}\right) + 2\bar{\phi}_1\left(\ell_{[a}n_{b]} + m_{[a}\bar{m}_{b]}\right) \tag{22}$$

$$2\left(\ell_{[a}n_{b]} + \bar{m}_{[a}m_{b]}\right) = \begin{pmatrix} 0 & 0 & iA & ip^2A \\ 0 & 0 & q^2A & -A \\ -iA & q^2A & 0 & 0 \\ -ip^2A & A & 0 & 0 \end{pmatrix}, \text{ with } A = \frac{(p+q)^2}{1+p^2q^2} \tag{23}$$

Conclusions

- Non-twisting N & D, $(T\&\not{T})$

$$\Psi_{ABCD} \Leftrightarrow \tilde{\nabla}_a F^{ab} = 0, \tilde{\nabla}^a \tilde{\nabla}_a S - \frac{1}{6}\tilde{R}S = 0 \tag{a}$$

- Non-twisting N & D, $(T\&\not{T})$

$$S^{(0)}_{12} \sim S^{(2)}_{12} \sim S \tag{b}$$

- non-twisiting N,

S $\Leftrightarrow$the geometrical properties of the wave surfaces of GWs:
the sources of the GWs are time-like, null, or space-like (c)
for RTN(Λ) (weak field limit)

Outlook

- More accurate connections between (the source of) gravitational waves and the double copy; e.g. RT sols with non-vanishing q !
- S in KN(Λ) class? - (α, β) !
- Higher dimensional case-NP formalism !!!!
- Einstein-Maxwell fields !!

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Thanks for your attention!

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