Rotational tidal Love numbers and their impact on neutron star inspirals

Phys. Rev. D 104, 044052
arXiv 2204.12510 (accepted for PRD)

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Relativistic tidal Love numbers - Static background

$$g_{00} = g_{00}^{(0)} + \sum_l g_{00}^{(l,\text{tidal})}\mathcal{E}^{(l)} + g_{00}^{(l,\text{response})} k_l^{\text{el}} \mathcal{E}^{(l)}$$

$$g_{00} = -1 + \frac{2M}{r} + \frac{3}{r^2} Q_{ij} n^{<i} n^{j>} + O\left(\frac{1}{r^2}\right) - \mathcal{E}_{ij} r^2 n^{<i} n^{j>} + O(r^2)$$

Induced multipole moments (adiabatic relations):

$$M_2 = \lambda_E^2 \mathcal{E}_0^{(2)}$$
$$S_3 = \lambda_M^3 \mathcal{B}_0^{(3)}$$

Love number definition

$$\lambda_E^{(l)} = \frac{\partial M_l}{\partial \mathcal{E}_0^{(l)}}, \quad \lambda_M^{(l)} = \frac{\partial S_l}{\partial \mathcal{B}_0^{(l)}}$$

Effects of rotation

For a slowly-rotating compact object:

$$g_{00} = g_{00}^{(0)} + (...)\mathcal{E}^{(2)} + g_{00}^{(2,\chi,\text{tidal})}\chi\mathcal{B}^{(3)} + g_{00}^{(2,\chi,\text{response})}\lambda_E^{(23)}\chi\mathcal{B}^{(3)}$$

$$g_{0\phi} = g_{0\phi}^{(0)} + (...)\mathcal{B}^{(3)} + g_{0\phi}^{(3,\chi,\text{tidal})}\chi\mathcal{E}^{(2)} + g_{0\phi}^{(3,\chi,\text{response})}\lambda_M^{(32)}\chi\mathcal{E}^{(2)}$$

Induced multipole moments:

$$M_2 = \lambda_E^2\mathcal{E}_0^{(2)} + \chi\lambda_E^{(23)}\mathcal{B}_0^{(3)}, \quad M_3 = \lambda_E^3\mathcal{E}_0^{(3)} + \chi\lambda_E^{(32)}\mathcal{B}_0^{(2)}$$

$$S_2 = \lambda_M^2\mathcal{B}_0^{(2)} + \chi\lambda_M^{(23)}\mathcal{E}_0^{(3)}, \quad S_3 = \lambda_M^3\mathcal{B}_0^{(3)} + \chi\lambda_M^{(32)}\mathcal{E}_0^{(2)}$$

RTLNs in a Lagrangian formulation

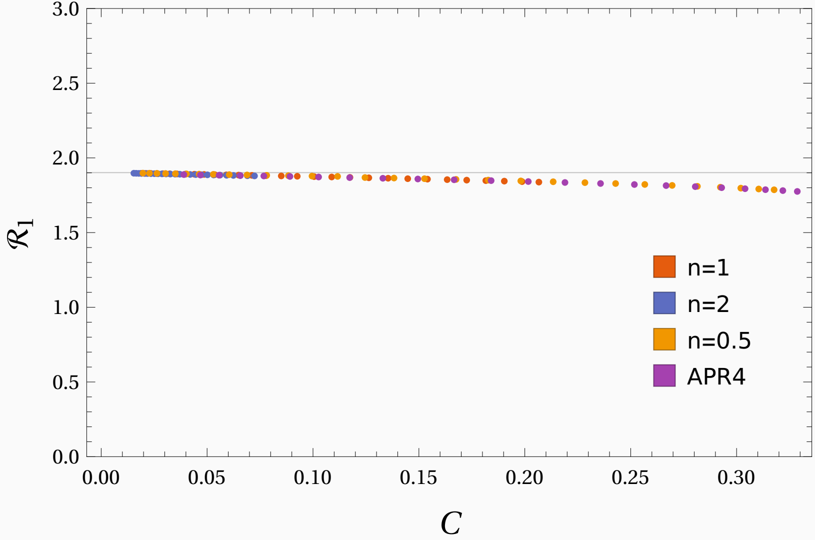
Phys. Rev. D 98, 104046 (2018), Abdelsalhin et al

- Impact of RTLNs in the GW phase at 6.5 PN order
- Studied via a Lagrangian formalism:

$$\mathcal{L}_2^{\rm int} = -\frac{1}{4\lambda_2} Q^{ab} Q^{ab} - \frac{1}{12\lambda_3} Q^{abc} Q^{abc} - \frac{1}{6\sigma_2} S^{ab} S^{ab} - \frac{1}{16\sigma_3} S^{abc} S^{abc} + \alpha J_2^a Q^{bc} S^{abc} + \beta J_2^a S^{bc} Q^{abc}\,,$$

- Simplification of adiabatic relations
    - $\sigma_{32} = 8\lambda_2\sigma_3\alpha$, $\lambda_{23} = 2\lambda_2\sigma_3\alpha$
    - $\lambda_M^{(32)}/\lambda_E^{(23)} = c^{\rm st}$

Ratio between RTLNs



Gravitational waveform

$$\tilde{h}(f) = \mathcal{A} f^{-\frac{7}{6}} e^{i\psi}, \quad \psi(f) = 2\pi f t_c - \phi_c - \frac{\pi}{4} + \frac{3}{128\eta x^{\frac{5}{2}}} \left(\psi_{\mathsf{pp}} + \psi_{\mathsf{T}}\right)$$

$$\begin{aligned}\psi_{\mathsf{T}} = &-\frac{39}{2}\tilde{\Lambda} x^5 + \left(-\frac{3115}{64}\tilde{\Lambda} + \delta\frac{6595}{364}\delta\tilde{\Lambda} + \tilde{\Sigma}\right) x^6 \\ &+ \left(\hat{\Lambda} + \hat{\Sigma} + \hat{K} + \hat{\Gamma}\right) x^{\frac{13}{2}}\end{aligned}$$

- $\hat{\Lambda}$, $\hat{\Sigma}$: spin-TLNs
- $\hat{K}$: tidal tail
- $\hat{\Gamma}$: RTLNs

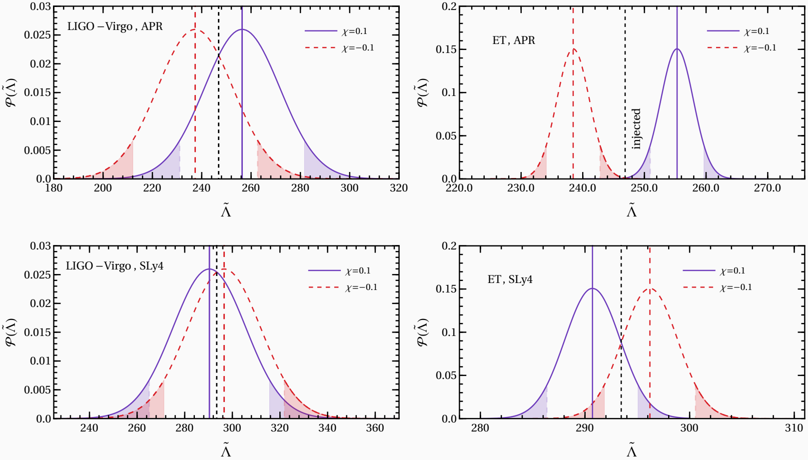
Impact of RTLNs in measurement of $\tilde{\Lambda}$

$$\mathcal{M}(h(\gamma_0), h_T(\gamma_T)) = \max_{\theta_0,\theta_T} \frac{(h|h_T)}{\sqrt{(h|h)\,(h_T|h_T)}}$$

$$p(\gamma_T) \propto \exp\left[-\rho^2\left(1 - \mathcal{M}(h|h_T)\right)\right]$$

- h: "true" signal
- h_T: template signal

Effect of spin-tidal terms



Measurability of $\hat{\Gamma}$

	EoS	χ_1	χ_2	$\tilde{\Lambda}$	$\hat{\Gamma}$
$\chi = 0.1$	APR4	$0.10^{+0.06}_{-0.12}$	$0.10^{+0.19}_{-0.10}$	550^{+80}_{-90}	$7.7^{+1.8}_{-2.0} \times 10^5$
	SLy4	$0.10^{+0.06}_{-0.10}$	$0.10^{+0.17}_{-0.10}$	690^{+90}_{-90}	$8.4^{+1.9}_{-1.9} \times 10^5$
$\chi = 0.05$	APR4	$0.05^{+0.09}_{-0.11}$	$0.05^{+0.18}_{-0.14}$	550^{+90}_{-90}	$3.9^{+2.0}_{-1.6} \times 10^5$
	SLy4	$0.05^{+0.10}_{-0.10}$	$0.05^{+0.16}_{-0.15}$	690^{+100}_{-90}	$4.2^{+2.2}_{-1.6} \times 10^5$

But spins and RTLNs entangled:

$$\hat{\Gamma} = \frac{\chi_1}{M^4}\left[\left(856\eta_1 - 816\eta_1^2\right)\lambda_{23}^{(1)} - \left(\frac{833\eta_1}{3} - 278\eta_1^2\right)\sigma_{23}^{(1)} - \nu\left(272\lambda_{32}^{(1)} - 204\sigma_{32}^{(1)}\right)\right] + (1 \leftrightarrow 2)\,.$$

Conclusions

- Hidden symmetry between RTLNs
- 6.5PN tidal terms (RTLNs included) necessary for accurate measurement of tidal deformability $\tilde{\Lambda}$
- More accurate waveform templates are necessary; Dynamical TLNs?