Cosmological Standard Timers in Primordial Black Hole Scenarios

Qianhang Ding
The Hong Kong University of Science and Technology

With Yi-Fu Cai (USTC), Chao Chen (HKUST), Yi Wang (HKUST)

GR23 July 2022
How to measure the Universe?
Standard Candle

\[ F = \frac{L}{4\pi d_L^2(z)} \]

Standard Ruler

\[ \theta = \frac{r_s}{D_M(z)} \]
Another way to measure the Universe?
How to know the elapsed time in the timer?
Initial state

Final state

Internal dynamics

\[
\frac{dh}{dt} = f(h)
\]
How to obtain $a(t)$?

$$1 + z(t) = \frac{a_0}{a(t)}$$
Standard timers in dynamical systems

Initial state

Initial state

Final state

Dynamics

Dynamics

Observed state
The primordial mass function of PBHs

\[ n(M) = \frac{f_{\text{PBH}}}{\sqrt{2\pi}\sigma M} \exp \left[ -\frac{\ln^2(M/M_{pk})}{2\sigma^2} \right] \]
How to extract the physical evolution time?
The evolution of the PBH mass function

\[ n(M; t) = \frac{dN}{dM} = \frac{dN}{dM_i} \frac{dM_i}{dM} = n(M; t_i) \frac{dM_i}{dM} = n(M; t_i) \frac{dM_i}{dM} \]

\[ \frac{dM}{dt} = -\frac{\alpha}{M^2} \implies M^3 = M_i^3 - \delta^3(\Delta t) \]

\[ n(M; t) = n(M; t_i) \frac{dM_i}{dM} = n(M; t_i) \frac{M^2}{(M^3 + \delta^3(\Delta t))^{2/3}} \]

\[ n(M; t) \approx \frac{n(\delta(\Delta t); t_i)}{\delta^2(\Delta t)} M^2, \quad M \ll \delta(\Delta t) \]
Can we see them?
Fermi LAT

$M_{\text{boson}} \text{ [g]}$

$M_{\text{pk}} \text{ [g]}$

Yi-Fu Cai, Chao Chen, Qianhang Ding & Yi Wang
2105.11481
How to extract the redshift from the observable?
Primary Hawking radiation from the PBH clustering

\[ P(E) = \int_0^\infty H_p(E, M)n(M)dM, \]

\[ H_p(E, M) = \frac{1}{2\pi} \frac{\Gamma_1(E, M)}{e^{8\pi G M E} - 1} \Gamma_1(E, M) \propto \begin{cases} G^4 M^4 E^4, & E < (8\pi G M)^{-1} \\ G^2 M^2 E^2, & E > (8\pi G M)^{-1} \end{cases} \]

Redshift in the observed photon flux

\[ F(E; z) = \frac{L(E(1 + z); z)}{4\pi d_L^2(z)} \approx \frac{(1 + z)^2 E^2 V}{4\pi d_L^2(z)} \int_0^\infty H_p(E(1 + z), M)n(M; z)dM \]

\[ H_p(E(1 + z), M) = H_p(E, M(1 + z)) \]

\[ \frac{4\pi F(E; z)}{E^2} \approx \int_0^\infty H_p(E, M')n\left(\frac{M'}{1 + z}; z\right) \frac{(1 + z)V}{d_L^2(z)} dM' \]
Redshift from the inverse problem

\[ P(E) = \int_{0}^{\infty} K(E, M)f(M)dM \Rightarrow f(M) = \int_{0}^{\infty} K^{-1}(E, M)P(E)dE \]

\[ \frac{4\pi F(E; z)}{E^2} \approx \int_{0}^{\infty} H_p(E, M)n\left(\frac{M}{1 + z}; z\right)\frac{(1 + z)V}{d^2_L(z)}dM \]

\[ f(M) \approx \int_{0}^{\infty} H^{-1}_p(E, M)\frac{4\pi F(E; z)}{E^2}dE \]

\[ f(M) = n\left(\frac{M}{1 + z}; z\right)\frac{(1 + z)V}{d^2_L(z)} \]
\[ n(M) = \frac{f_{\text{PBH}}}{\sqrt{2\pi \sigma M}} \exp \left[ -\frac{\ln^2(M/M_{pk})}{2\sigma^2} \right] \]

\[ \tilde{n}(M; z) = n \left( \frac{M}{1+z} ; z \right) \]
The initial probability distribution on $a$ and $e$

$$\frac{dP}{d\alpha de} = \frac{3}{4} f_{PBH} \frac{a^{1/2}}{\bar{x}^{3/2}} \frac{e}{(1-e^2)^{3/2}}$$
How to extract the physical evolution time?
The evolution of probability distribution in PBH binaries

\[
\frac{dP}{da_t de_t} = \frac{dP}{da_i de_i} \det J(a, e, \Delta t)
\]

\[J(a, e, \Delta t) = \begin{pmatrix}
\frac{\partial a_i}{\partial a_t} & \frac{\partial a_i}{\partial e_t} \\
\frac{\partial e_i}{\partial a_t} & \frac{\partial e_i}{\partial e_t}
\end{pmatrix}
\]

\[
\frac{da}{dt} = -\frac{128}{5} \frac{G^3 M_{\text{PBH}}^3}{c^5 a^3 (1 - e^2)^{7/2}} \left( 1 + \frac{73}{24} e^2 + \frac{37}{96} e^4 \right)
\]

\[
\frac{de}{dt} = -\frac{608}{15} \frac{G^3 M_{\text{PBH}}^3}{c^5 a^4} \frac{e}{(1 - e^2)^{5/2}} \left( 1 + \frac{121}{304} e^2 \right)
\]
Can we see them?
How to extract the redshift from the observable?
Redshifted Chirp Mass

\[ M_z = (1 + z) M \]

\[ M_{z_3} = (1 + z_3)M \]
\[ M_{z_2} = (1 + z_2)M \]
\[ M_{z_1} = (1 + z_1)M \]
\[
\frac{dP}{da_Z} = \frac{1}{1 + z \frac{dP}{da_i}}
\]
\[ \Delta t = \int_{z_1}^{z_2} \frac{dz}{H(z)(1 + z)} \]

\[ H(z) = H_0 \sqrt{\Omega_\gamma (1 + z)^4 + \Omega_m (1 + z)^3 + \Omega_\Lambda} \]
Thank you!