Testing Geometric Surface Conjecture for Rotating Transversable Wormholes

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Outline

- General Metric for Rotating Transversable Wormholes (E. Teo 1998).
- Geometric Surface Conjecture (McNutt, Julius, EUCOS, 2021)
- Classification Issues with Teo (McNutt, Julius, EUCOS 2022)

Edward Teo's Wormhole

- Morris-Thorne (Spherical Symmetry):

$$ds^2 = -e^{\Phi(r)}dt^2 + \left(1 - \frac{b(r)}{r}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2)$$

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- Teo (Axi-Symmetric Generalization):

$$ds^2 = -N(r,\theta)dt^2 + (1 - \frac{b(r,\theta)}{r})^{-1} dr^2 + r^2 K(r,\theta)^2[d\theta^2 + \sin^2(\theta)(d\phi + \omega(r,\theta)dt)^2]$$

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- Flair out condition of throat

$$\frac{d^2r}{dz^2} = \frac{b(r,\theta) - r\partial_r b(r,\theta)}{2b(r,\theta)^2} > 0$$

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- Algebraically special subcases have not been found
- Two non-orthogonal commuting killing vectors:

$$X_1 = \partial_t \qquad X_2 = \partial_\phi$$

$$X_1 \cdot X_2 = g_{t\phi} = 2r^2 K^2(r,\theta)\omega(r,\theta)\sin^2(\theta)$$

Geometric Horizon Conjecture

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- Spacetime horizons of a black hole or wormhole throat are always more algebraically special than other regions of the spacetime
- Therefore, the horizon can be detected locally using a vanishing of scalar curvature invariants.
- More elegant, foliation-independent

Issues with Geometric Horizons

- Cartan invariants, which require a specific canonical co-frame
- Type I Example: $\Psi_0 = 1 \; \Psi_1 = \Psi_3 = 0 \;\; \Psi_2 \neq 0 \;\; \Psi_4 \neq 0$

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- Example: event horizon and wormhole throat
- Also between Killling horizons, cosmological horizons, Kerr ergospheres

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- Well tested for Type D/II spacetimes, how about Type I?
- We'll see for Teo the result is suprising

Geometric Surface Conjecture

- In complex-null frame : $(l, n, m, \bar{m})$
- Wormhole Throat (Hochberg-Visser '98):

$$\theta_{(l)} = 2m^a \bar{m}^b \nabla_b l_a = 0 \qquad l^a \nabla_a \theta_{(l)} \geq 0$$

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- Apparent Horizon (Hayward '94):

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$$\mu = \frac{C_{1231;4}}{3\Psi_2}$$

Teo Example

- Wormhole throat: $\rho = 0 \quad D\rho \leq 0$
- Apparent horizon: $\rho = 0 \quad \mu > 0 \quad \Delta\rho > 0$
- Canonical frame $\rho = \mu = 0$, everywhere

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- Canonical frame $\rho = \mu = 0$, everywhere
- Non-canonical frame:

$$n = -\frac{|N|}{\sqrt{2}}dt - \frac{r(1-\partial_r b)}{\sqrt{2r(r-b)(\partial_r b - 1)^2}}dr$$

$$l = -\frac{|N|}{\sqrt{2}}dt + \frac{r(1-\partial_r b)}{\sqrt{2r(r-b)(\partial_r b - 1)^2}}dr$$

$$m = -\frac{ir\sin(\theta)K\omega}{\sqrt{2}}dt + \frac{ir^{3/2}}{\sqrt{2}\sin(\theta)^3}|\partial_r b - 1|K^2\sqrt{\frac{\sin(\theta)(r-b)}{r(r-b)K^2(\partial_r b - 1)^2}}d\theta + \frac{ir\sin(\theta)K}{\sqrt{2}}d\rho$$

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- Spin Coefficients at the throat:

$$\mu = \rho = \lim_{r \to b} sgn(\partial_r b(r) - 1) \frac{\sqrt{r - b(r)}(K(r,\theta) + r\partial_r K(r,\theta))}{\sqrt{2} r^{\frac{3}{2}} K(r,\theta)} = 0$$

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- It is required that: $K(b,\theta) \geq |b\partial_r K(b,\theta)|$ since $K(r,\theta) > 0$
- Will hold when $K(r,\theta) = a(\theta) r^n$ for n>-1

Morals

- In this case this frame is far from a canonical CK frame as all Weyl components are non-vanishing
- If we move to a (nearly) canonical frame we see an issue emerges: ρ and μ vanish everywhere
- In this case our scheme of writing ρ and μ

In terms of Cartan invariants breaks down.

Invariant Conditions

$$I_2 = *C_{abcd}C^{abcd} = 0$$

- C_{abcd} is the Weyl tensor
- A suggestive step towards invariant classification
- Unclear how to write these conditions in terms of scalar invariants since they are quite complicated
- Detecting flair out is very problematic

Equivalence and subclassification

- A related but distinct issue is that because both scalar invariants and CK invariants are so complicated determining inequivalence of solutions can be difficult
- In particular, if two solutions are both of type I with the same scalar invariants nonvanishing the equations required to be solved are very complicated

Conclusion/ Needs

- New way to invariantly detect physically significant surfaces

- Improvement on Cartan-Karlhede algorithm

Thank you !!!

Questions?