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Compact Star in General F(R) Gravity

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"Compact star in general F(R) gravity: Inevitable degeneracy problem and non-integer power correction"

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F(R) gravity theory

Action of F(R) gravity

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}F(R)$$

[Buchdahl (1970)]

cf.) EH-action

$$\int d^4x\sqrt{-g}R$$

Replace: $R \to F(R)$

EOM with matter field

$$F_R(R)R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}F(R) + \left(g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu\right)F_R(R) = \kappa^2 T_{\mu\nu}$$

Here, $F_R = \partial_R F(R)$

- F(R) gravity includes fourth-order derivative of metric

Trace of the EOM

$$\Box F_R(R) = \frac{1}{3}\kappa^2 T + \frac{1}{3}\left[2F(R) - F_R(R)R\right]$$

- The Ricci scalar is dynamical: scalar mode $\phi = F_R(R)$

Background & Research Purpose

Compact star in F(R) gravity

There are lots of earlier works...

- Rotating, With magnetic field etc.
- Case study for various EOS
- Case study for various F(R) models

Question:
How do we distinguish two ambiguities?

Motivation:

Properties of Compact Star in general F(R) gravity?

- How can we extract signals of modified gravity from astrophysical observation?
- How can we constrain F(R) function separately from hadron physics ?
- How can handle fourth order of derivative? What kind of boundary conditions?

Rudiments

EOM

$$F_R(R)R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}F(R) + \left(g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu\right)F_R(R) = \kappa^2 T_{\mu\nu}$$

Generalized Bianchi identity

$$\nabla^\mu\left(\frac{1}{2}g_{\mu\nu}F - R_{\mu\nu}F_R - g_{\mu\nu}\Box F_R + \nabla_\mu\nabla_\nu F_R\right) = 0$$

Static and Spherically Symmetric metric

$$ds^2 = \sum_{\mu,\nu=t,r,\theta,\phi} g_{\mu\nu}dx^\mu dx^\nu = -\,\mathrm{e}^{2\nu(r)}dt^2 + \mathrm{e}^{2\lambda(r)}dr^2 + r^2\sum_{i,j=\theta,\phi}\bar{g}_{ij}dx^i dx^j\,,$$

$$\sum_{i,j=\theta,\phi}\bar{g}_{ij}dx^i dx^j = d\theta^2 + \sin^2\theta\, d\phi^2\,.$$

Perfect fluid as internal matter

$$T_{tt} = -g_{tt}\rho = \mathrm{e}^{2\nu}\rho\,, \quad T_{rr} = g_{rr}p = \mathrm{e}^{2\lambda}p\,, \quad T_{ij} = g_{ij}p = r^2\bar{g}_{ij}p$$

ρ and p are total energy density and pressure.

To solve TOV equation

We assume density profile $\rho = \rho(r)$ and EOS $p = p(\rho)$

$\rightarrow p = p(r)$

Note) M-R relation is given by integration of density profile $\rho = \rho(r)$

TOV equation

$$
\begin{cases}
\nu'(\rho + p) + \dfrac{dp}{dr} = 0 \quad \Rightarrow \quad \nu = \nu(r) \\
-\kappa^2(\rho + p) = -\dfrac{2(\nu' + \lambda')}{r} \mathrm{e}^{-2\lambda} F_R + \mathrm{e}^{-2\lambda} \left[F_R'' - (\nu' + \lambda') F_R' \right] \\
-\kappa^2(\rho + p) = -\left\{ \dfrac{1}{r^2} + \mathrm{e}^{-2\lambda} \left[\nu'' + (\nu' - \lambda')\nu' + \dfrac{\nu' + \lambda'}{r} - \dfrac{1}{r^2} \right] \right\} F_R - \mathrm{e}^{-2\lambda} \left(\nu' - \dfrac{1}{r} \right) F_R'
\end{cases}
$$

Reconstruction of F(R) Function

F(R) as solution of TOV equation

Defining $N(r) \equiv e^{-2\nu-2\lambda}$ and rewriting two eqs.

$$\begin{cases} N'\left(\frac{F_R}{r}+\frac{1}{2}F_R'\right)+NF_R''=\underbrace{-e^{-2\nu}\kappa^2(\rho+p)}_{\text{known}} \\ -\left[N\left(\nu''+2\nu'^2-\frac{1}{r^2}\right)+N'\left(\frac{\nu'}{2}-\frac{1}{2r}\right)\right]F_R-N\left(\nu'-\frac{1}{r}\right)F_R'=e^{-2\nu}\left[-\kappa^2(\rho+p)+\frac{1}{r^2}F_R\right] \end{cases}$$

$\Rightarrow N = N(F_R, r)$

We find differential eq. for $F_R(r)$

$$E\left(F_R'', F_R', F_R, \underbrace{\nu''(r), \nu'(r), \nu(r), \rho(r), p(r)}_{\text{known}}\right)=0$$

$\Rightarrow F_R = F_R(r)$

From $N(r) \equiv e^{-2\nu-2\lambda}$, we find $\lambda = \lambda(r)$

Ricci scalar $R = R(\nu'', \nu', \nu, \lambda', \lambda, r)$ gives $r = r(R)$

Finally, we obtain $F_R = F_R(r) \to F = F(R)$

Reconstruction of F(R) Function

Degeneracy Problem

If M-R relation (= density profile) and EOS are given,
we can solve TOV eq. w.r.t. F(R) function.

In other words, even if M-R relation is determined by observations, we can find a model of F(R) gravity for arbitrary EOS.

Degeneracy between EOS and F(R) function

- M-R relation is not conclusive enough to separately constrain EOS and modified gravity...
- Note) F(R) dof --> scalar mode dof. = choice of potential

Constraint from Boundary Condition

Near surface of compact star

One can reconstruct F(R) function to lead to a given M-R relation.

But, it does not mean any F(R) function is allowed.

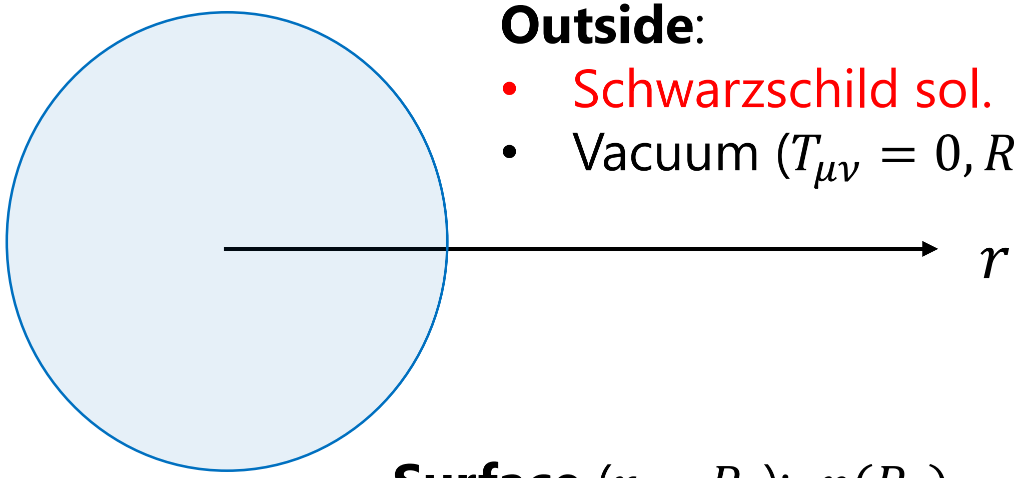
Let's consider following setup

Outside:
- Schwarzschild sol.
- Vacuum ($T_{\mu\nu} = 0, R = 0$)

Inside:
- TOV eq.
- Energy-Polytrope

$$p = K\rho^{1+\frac{1}{n}}$$

- $0.5 < n < 1$ for NS



Surface ($r = R_s$): $p(R_s) = 0$

Constraint from Boundary Condition

Near surface of compact star

Because EOM include second-order derivative of Ricci scalar, R'', R and R' must be continuous at surface.

Thus, near the surface, R behaves like

$$R \sim R_0 \left(1 - \frac{r}{R_s}\right)^{\alpha} \quad (\alpha \geq 2)$$

Although pressure p should vanish at $r = R_s$,
its convergent behavior is unknown.

So, using Taylor expansion near the surface, we assume $p(r)$ behaves like

$$p(r) \sim p_0 \left(1 - \frac{r}{R_s}\right)^m \left[1 + p_1 \left(1 - \frac{r}{R_s}\right) + p_2 \left(1 - \frac{r}{R_s}\right)^2 + \cdots\right]$$

Constraint on F(R) Function

Continuity at surface

Conservation law

$$\nu(r) = -\int^{p(r)} \frac{dp}{\tilde{K}p^{1+\frac{1}{n}} + p} \sim \nu_0 - (1+n)\ln\left\{1 + \nu_1\left(1 - \frac{r}{R_s}\right)^{\frac{m}{1+n}}\left[1 + p_1\left(1 - \frac{r}{R_s}\right) + p_2\left(1 - \frac{r}{R_s}\right)^2 + \cdots\right]\right.$$

$$\left(\nu_1 \equiv K^{\frac{1}{1+\frac{1}{n}}} p_0^{\frac{1}{1+n}}\right)$$

Continuity of $\nu(r)$: $\nu_0 = \frac{1}{2}\ln\left(1 - \frac{2M}{R_s}\right)$ (connecting to Sch. sol.)

Continuity of $\nu'(r)$: $m = n + 1$

And

$$\frac{M}{R_s} = \frac{1}{2}\left[1 - \frac{1}{1 + 2(n+1)K^{\frac{1}{1+\frac{1}{n}}} p_0^{\frac{1}{1+n}}}\right] \Leftrightarrow 0 < \frac{M}{R_s} < \frac{1}{2}$$

Note) Above follows from behavior near the surface and doesn't depend on details of inner structure.

Constraint on F(R) Function

Continuity at surface

Continuity of $\nu''(r)$:

$$p_1 = \frac{(1+n)\nu_1}{2} + \frac{1}{\nu_1}\left[\frac{\frac{M}{R_s}}{1-\frac{2M}{R_s}} + \frac{\frac{M^2}{R_s^2}}{\left(1-\frac{2M}{R_s}\right)^2}\right]$$

Continuities of higher-order deriv. give higher-order coeff. of pressure $p(r)$ → Determines $\nu(r), p(r)$

Continuity conditions tell us

$$N = 1 + \mathcal{O}\left(\left(1-\frac{r}{R_s}\right)^{N_0}\right) \quad (3 \leq N_0 \in \mathbb{N})$$

$$p \sim p_0\left(1-\frac{r}{R_s}\right)^{n+1}, \quad \rho \sim \tilde{K} p_0^{\frac{n}{n+1}}\left(1-\frac{r}{R_s}\right)^{n}, \quad \rho + p \sim \rho \sim \left(1-\frac{r}{R_s}\right)^{n}$$

Constraint on F(R) Function

Non-Integer power correction

Assuming F(R) function describes corrections to GR as

$$F_R \sim F_0 + F_1 \left(\frac{R}{R_0}\right)^{\beta} \sim F_0 + F_1 \left(1 - \frac{r}{R_s}\right)^{\alpha\beta}$$

From EOM

$$\underline{N'\left(\frac{F_R}{r} + \frac{1}{2}F_R'\right)} + \underline{NF_R''} = \underline{-\mathrm{e}^{-2\nu}\kappa^2(\rho + p)}$$

$$\sim \left(1 - \frac{r}{R_s}\right)^{N_0 - 2} \qquad \sim \left(1 - \frac{r}{R_s}\right)^{\alpha\beta - 2} \qquad \sim \left(1 - \frac{r}{R_s}\right)^{n}$$

We find 1st term in LHS cannot balance with RHS,
and thus, 2nd term in LHS balance with RHS.

$$\alpha\beta - 2 = n \Leftrightarrow \beta = \frac{n+2}{\alpha} \notin \mathbb{N} \qquad F(R) \sim \Lambda + F_0 R + \underline{\frac{F_1 R_0}{\beta + 1}\left(\frac{R}{R_0}\right)^{\beta + 1}} + \cdots$$

Non-integer power shows up

Conclusion

Degeneracy problem

- F(R) function, M-R relation (=density profile), EOS
- Even if one of three is given, the other two are degenerated.
- Only with M-R relation, we cannot constrain F(R) gravity and EOS separately
- Known as Reconstruction technique in Cosmology

[Nojiri et al., PLB 681 (2009)]

So, we need other observational data

- Tidal deformation (perturbed TOV eq.)
- Thermal evolution (cooling) etc.

Non-integer power correction

- $F(R) = R + \alpha R^{1+\beta}$ $(0 < \beta < 1)$ for DE or Inflation

[Artymowski & Lalak, JCAP 09 (2014)] etc.

- Our approach gives us guideline for model-building
- Ext. Sch. Sol. requires that scalar field is screened

⇔ Chameleon mechanism [Khoury & Weltman, PRL 93 (2004)]