Gowdy models as physically viable inflationary early spacetimes

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Overview

- Most of the models of the early universe are homogeneous and isotropic. Planck observations (2018) did not confirm with strong evidence any departure.

- However there is consensus that some anomalies at large scales (dipolar, quadrupolar, etc.) are present, indicating new (pre-)inflationary physics.

- We have studied the influence of primordial spacetime anisotropies (Agullo, O., Sreenath, Wilson-Ewing — 2003.02304, 2003.08428, 2006.01883, 2206.04037).

- We have found that
  a) they generate quadrupolar anomalies in the CMB, which can be constrained by Planck data,

  b) and nontrivial *TT*, *TE*, *EE*, *BB*, *TB* and *EB* off-diagonal components (they vanish in the isotropic limit).

Overview

- However those models do not violate parity, e.g., they do not generate dipolar anomalies.
- One way is to include high-order perturbative corrections and their backreaction (non-Gaussianities).
- Another way (the aim here) is to explore the influence of very early nonperturbative inhomogeneities (and anisotropies) in the physics of the CMB. We expect they will generate further large scale anomalies.
- Polarized Gowdy cosmologies is one of the simplest settings where we can explore those effects.
- In this talk we will briefly introduce the model and discuss some of the consequences of nonperturbative inhomogeneity.

Introduction

- We start with the classical ADM formulation of Einstein-Hilbert action minimally coupled to a scalar field on the T^3.

$$\left\{\Phi(\vec{x}), P_\Phi(\vec{x}')\right\} = \delta^{(3)}(\vec{x}-\vec{x}'), \quad \left\{h_{ij}(\vec{x}), \pi^{kl}(\vec{x}')\right\} = \delta^k_{(i}\delta^l_{j)}\delta^{(3)}\left(\vec{x}-\vec{x}'\right),$$

- The total Hamiltonian is a linear combination of constraints

$$H = \int \mathrm{d}^3x \left[N(\vec{x})\mathbb{S}(\vec{x}) + N^i(\vec{x})\mathbb{V}_i(\vec{x})\right],$$

with constraints given by

$$\mathbb{S}(\vec{x}) = \frac{2\kappa}{\sqrt{h}}\left(\pi^{ij}\pi_{ij} - \frac{1}{2}\pi^2\right) - \frac{\sqrt{h}}{2\kappa}{}^{(3)}R + \frac{1}{2\sqrt{h}}P_\Phi^2 + \sqrt{h}V(\Phi) + \frac{\sqrt{h}}{2}D_i\Phi D^i\Phi \approx 0,$$

$$\mathbb{V}_i(\vec{x}) = -2\sqrt{h}h_{ij}D_k\left(h^{-1/2}\pi^{kj}\right) + P_\Phi D_i\Phi \approx 0.$$

T^3 Bianchi I spacetimes

- The spacetime metric has 3 space-like commuting KVFs, and it is given by,

$$ds^2 = -N_0^2 dt^2 + a_1^2 dx^2 + a_2^2 dy^2 + a_3^2 dz^2,$$

- The Poisson brackets in phase space

$$\{a_I, p_{a_J}\} = \frac{1}{L_x L_y L_z}\delta_{IJ}\,, \quad \{\phi_0, P_{\phi_0}\} = \frac{1}{L_x L_y L_z}\,,$$

- The reduced total Hamiltonian is a (global) constraint, $\underline{\mathcal{H}}^{(2)} = N_0 \underline{S}^{(2)}$

$$\underline{S}_0^{(2)} = \frac{L_x L_y L_z}{a_1 a_2 a_3}\left[\frac{\kappa}{4}\left(a_1^2 p_{a_1}^2 + a_2^2 p_{a_2}^2 + a_3^2 p_{a_3}^2 - 2a_1 p_{a_1} a_2 p_{a_2} - 2a_1 p_{a_1} a_3 p_{a_3} - 2a_2 p_{a_2} a_3 p_{a_3}\right)\right.$$
$$\left. + \frac{P_{\phi_0}^2}{2} + (a_1 a_2 a_3)^2 V(\phi_0)\right],$$

which amount to the Friedmann equation $H^2 = \frac{\kappa}{3}\rho_{\phi_0} + \frac{\sigma^2}{6}$ with

$$H = \frac{1}{3}\left(H_1 + H_2 + H_3\right), \quad \sigma^2 = (H_1 - H)^2 + (H_2 - H)^2 + (H_3 - H)^2, \quad H_i = \frac{\dot{a}_i}{a_i}.$$

T^3 polarized Gowdy reduced model

- The spacetime metric has 2 space-like commuting KVFs. After gauge fixing, it is given by

$$ds^2 = \left\{-[N_0^2 - a_1^2[N_1^x(x)]^2]dt^2 + 2a_1^2 N_1^x(x)dxdt + a_1^2 dx^2\right\} e^{X(x)} + a_2^2 e^{-\frac{\xi_1(x)}{\sqrt{a_2 a_3}}} dy^2 + a_3^2 e^{\frac{\xi_1(x)}{\sqrt{a_2 a_3}}} dz^2,$$

with $N_1^x(x)$ and $X(x)$ inhomogeneous phase space variables of

$$\begin{aligned} \{a_I, p_{a_J}\} &= \frac{1}{L_x L_y L_z}\delta_{IJ}\,, \quad \{\phi_0, P_{\phi_0}\} = \frac{1}{L_x L_y L_z}\,, \\ \{\xi_1(x), \pi_{\xi_1}(x')\} &= \frac{1}{L_y L_z}\delta(x-x')\,, \quad \{\phi_1(x), \pi_{\phi_1}(x')\} = \frac{1}{L_y L_z}\delta(x-x')\,, \end{aligned}$$

Linear inhomogeneous sector

- There is an interesting regime where the nonperturbative inhomogeneities satisfy linear differential equations
  a) When $V(\phi)$ is negligible (e.g. kinetically dominated sector).
  b) If $V(\phi)$ is non-negligible but $|X(x)| \ll 1$ for all x.
- In this regime, the Hamiltonian is quadratic in the inhomogeneities (linear EOMs). It amounts to the effective Friedmann equation

$$H^2 = \frac{\kappa}{3}\left(\rho_{\phi_0} + \rho_{\phi_1} + \rho_{\xi_1}\right) + \frac{\sigma^2}{6}.$$

$$\rho_{\phi_1} = \sum_{n\neq 0,1}\left(\frac{|\tilde{P}_{\phi_1}|^2}{2a_1^2a_2^2a_3^2} + \frac{1}{2a_1^2}k_n^2|\tilde{\phi}_1|^2 + \frac{1}{2}M^2|\tilde{\phi}_1|^2\right),$$

$$\rho_{\xi_1} = \sum_{k_n\neq 0}\left(\frac{\kappa}{a_1^2a_2a_3}|\tilde{\pi}_{\xi_1}|^2 + \frac{1}{4\kappa a_1^2a_2a_3}k_n^2|\tilde{\xi}_1|^2 - \kappa\frac{p_{a_1}^2}{16a_2^2a_3^2}|\tilde{\xi}_1|^2\right),$$

Linear inhomogeneous sector

- It is then convenient to expand the inhomogeneities in Fourier modes. For instance

$$\xi_1(x) = \sum_{k_n \neq 0} \tilde{\xi}_1(k_n) e^{ik_n x},$$

where $k_n = \frac{2n\pi}{L_x}$ with $n = 1, 2, \ldots, \infty$, $\quad \tilde{\xi}_1(k_n)^* = \tilde{\xi}_1(-k_n)$ and

$$\{\tilde{\xi}_1(k_n), \tilde{P}_{\xi_1}(k_{n'})\} = \frac{1}{L_x L_y L_z} \delta_{k_n, -k_{n'}}, \quad \{\tilde{\phi}_1(k_n), \tilde{P}_{\phi_1}(k_{n'})\} = \frac{1}{L_x L_y L_z} \delta_{k_n, -k_{n'}}.$$

- The modes of the inhomogeneous fields satisfy the Klein-Gordon equations

$$\ddot{\tilde{\xi}}_1(k_n) + H_1 \dot{\tilde{\xi}}_1(k_n) + \frac{k_n^2}{a_1^2} \tilde{\xi}_1(k_n) + \frac{\kappa^2 p_{a_1}^2}{4 a_2^2 a_3^2} \tilde{\xi}_1(k_n) = 0,$$

$$\ddot{\tilde{\phi}}_1(k_n) + 3H \dot{\tilde{\phi}}_1(k_n) + \frac{k_n^2}{a_1^2} \tilde{\phi}_1(k_n) + M^2 \tilde{\phi}_1(k_n) = 0,$$

Equations of motion

- Thus, we can express the Fourier modes of both inhomogeneities as

$$\tilde{\xi}_1(k_n,t) = a_{\tilde{\xi}_1,u}(k_n)u_{\tilde{\xi}_1}(k_n,t) + a^*_{\tilde{\xi}_1,u}(-k_n)u^*_{\tilde{\xi}_1}(-k_n,t),$$
$$\tilde{\phi}_1(k_n,t) = a_{\tilde{\phi}_1,v}(k_n)v_{\tilde{\phi}_1}(k_n,t) + a^*_{\tilde{\phi}_1,v}(-k_n)v^*_{\tilde{\phi}_1}(-k_n,t),$$

with $u_{\tilde{\xi}_1}(k_n,t)$ and $v_{\tilde{\phi}_1}(k_n,t)$ two complex solutions with unit norm with respect to the inner products

$$\langle {}^{(a)}u(k_n,t), {}^{(b)}u(k_n,t)\rangle = iL_xL_yL_z\left[{}^{(a)}\tilde{\xi}_1^*(k_n,t)^{(b)}\tilde{\pi}_{\xi_1}(k_n,t) - {}^{(a)}\tilde{\pi}^*_{\tilde{\xi}_1}(k_n,t)^{(b)}\tilde{\xi}_1(k_n,t)\right],$$
$$\langle {}^{(a)}v(k_n,t), {}^{(b)}v(k_n,t)\rangle = iL_xL_yL_z\left[{}^{(a)}\tilde{\phi}_1^*(k_n,t)^{(b)}\tilde{P}_{\phi_1}(k_n,t) - {}^{(a)}\tilde{P}^*_{\phi_1}(k_n,t)^{(b)}\tilde{\phi}_1(k_n,t)\right].$$

and $\{a^*_{\tilde{\xi}_1,u}(k_n), a_{\tilde{\xi}_1,u}(k_{n'})\} = i\delta_{k_n,k_{n'}}$ and $\{a^*_{\tilde{\phi}_1,v}(k_n), a_{\tilde{\phi}_1,v}(k_{n'})\} = i\delta_{k_n,k_{n'}}$ complex creation and annihilation variables (Dirac observables).

Initial data

- For the homogeneous sector, we set initial data in the limit in which inhomogeneities are negligible. Here, we specify:

  i) ϕ_0, H and the sign of P_{ϕ_0},

  ii) the amount of anisotropies σ^2 and how they are distributed between principal directions σ_1/σ,

  iii) we also choose a to be equal one at the initial time and the directional scale factors a_i coinciding at the end of inflation.

- We fix the inhomogeneous fields such that we study the influence of matter and geometry inhomogeneities independently, and such that they start within following configurations:

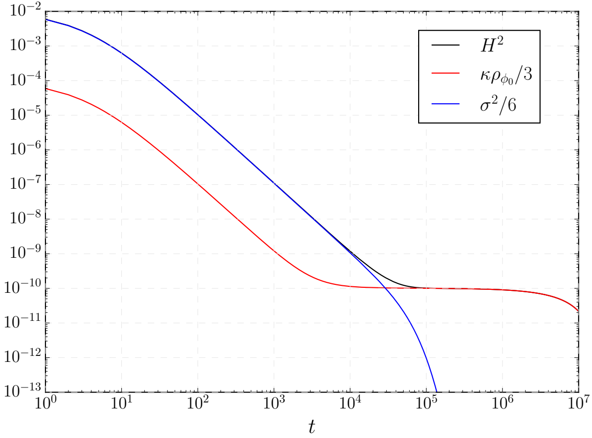
  a) 2-mode states and Gaussian (multi-mode) configurations,

  b) within each family, we choose them such that they are initially either, super-Hubble, sub-Hubble or of the order the Hubble scale,

  c) We also consider parity even - odd configurations, and always with $|a_{\tilde{\xi}_1,u}(k_n)| = |a_{\tilde{\xi}_1,u}(-k_n)|$.

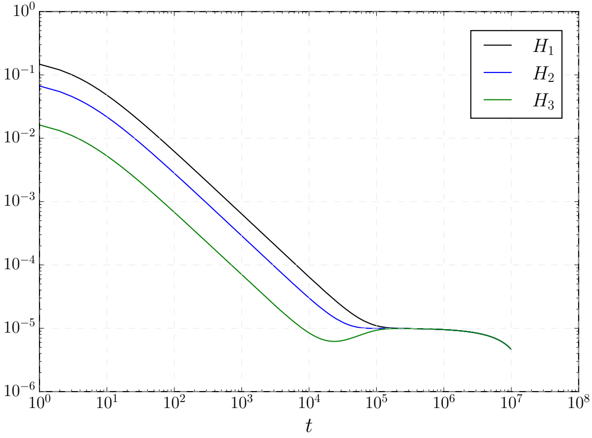
  The last condition c) guaranties that the homogeneous momentum constraint left in the reduced theory is satisfied.

Simulations: homogeneous model



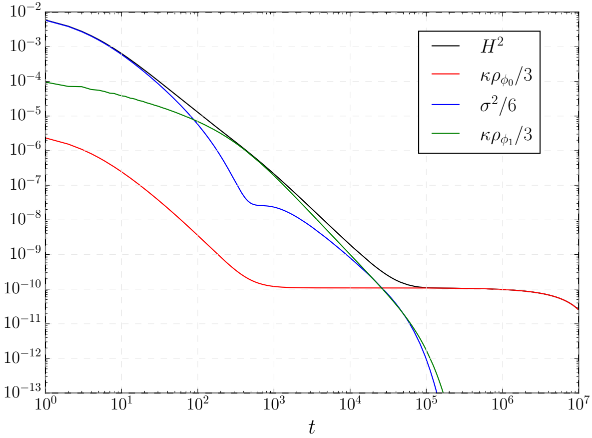
Components of the Friedmann equation.

Simulations: homogeneous model



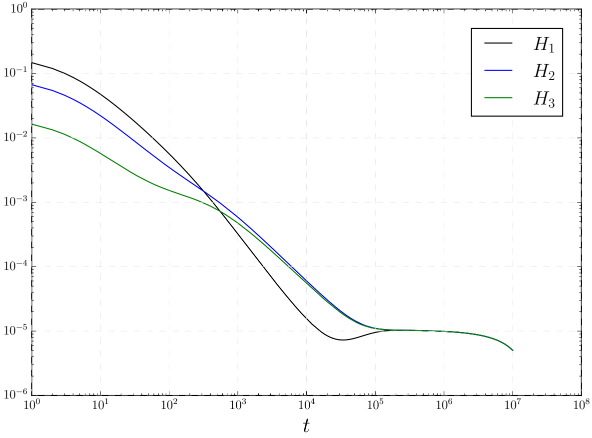
Directional Hubble rates.

Simulations



Some components of the Friedmann equation. This simulation corresponds to an anisotropic solution with two small scale large amplitude matter inhomogeneities $\phi_1(k_n)$ and no geometric inhomogeneities $\xi_1(k_n) = 0$. Here $X(x) < 0.1$ at all times.

Simulations: Gowdy with pure tensor inhomogeneities



Directional Hubble rates.

Summary

- We consider a polarized T^3 Gowdy model coupled to a massive scalar field.
- After gauge fixing, we show that the equations of motion of inhomogeneities are not linear due to the mass of the scalar field.
- We identify sectors where linearity is recovered: i) regimes where the potential energy of the inflaton is negligible (expected at early times) and ii) trajectories where the determinant of the spatial metric is nearly homogeneous (late time expansion).
- We provide an effective Friedmann equation and discuss the solution space of the linear equations of motion of the Fourier modes of the inhomogeneities.
- We solved the dynamics numerically and studied several inhomogeneous configurations.
- Preliminary studies show that nonperturbative inhomogenities contribute to the mean Hubble rate as a source of anisotropies at intermediate times. They dilute rapidly once slow-roll inflation begins.

Outlook

- Our next objective is to analyze the effects of large scale inhomogeneities on the CMB. Despite inhomogeneities dilute, they keep some memory!
- We expect that perturbations leaving the horizon around or a bit after the onset of slow-roll will feel the presence of non-perturbative inhomogeneities.
- We expect to obtain multipolar anomalies (dipolar, quadrupolar, ...) that can constrain the parameter space of the model.
- Extensions into the Planck regime, for instance, adopting loop quantum gravity techniques (Mena-Marugán, Garay, Martín-Benito, Martín-de Blas, Elizaga Navascués, Castelló Gomar, Fernández-Méndez, García-Quismondo, Prado, Neves, O., ...).