LORENTZIAN SPACETIMES FROM STOCHASTIC COMPOSITE GRAVITY

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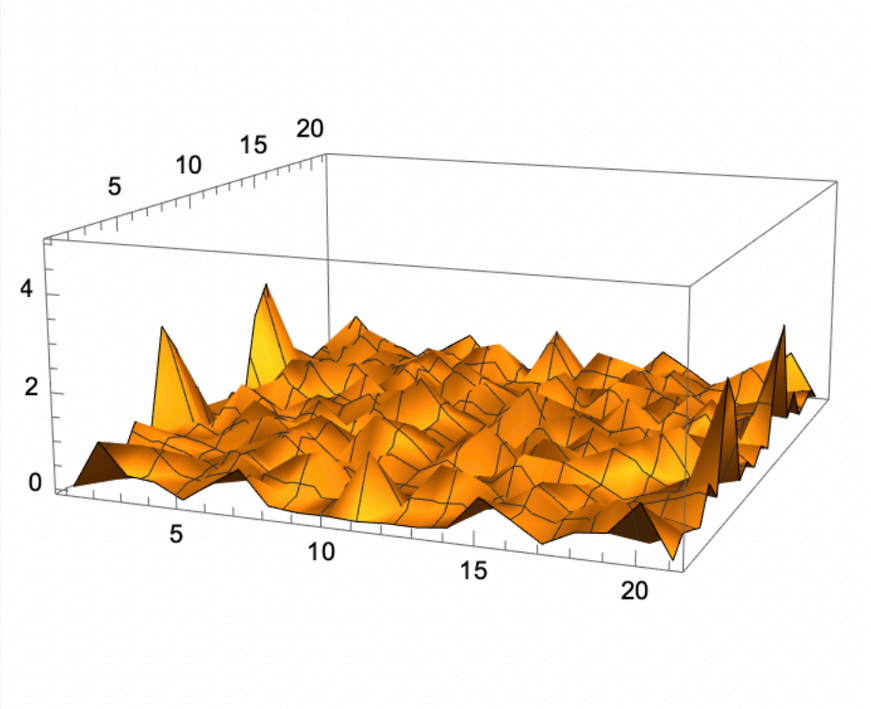
Stochastic Composite Gravity



Composite Gravity

A composite operator plays the role of the spacetime metric.

The expectation value of the operator provides a background spacetime.

Fluctuations about the vacuum provide gravitation, with Planck scale dependent on a physical ultraviolet regulator.

Stochastic Mechanics

Quantum Mechanics/QFT is described in terms of stochastic processes.

Discreteness of stochastic events provides an ultraviolet regulator.

Composite Gravity: A toy model

Akama, Chikashige, Terazawa (1970s);
C. Carone, JE, D. Vaman; + Claringbold; + S. Chaurasia, Y. Zhou (2010s-)

$$S = \int \mathrm{d}^D x \sqrt{|g|} \left[\frac{1}{2} g^{\mu\nu} \left(\sum_{a=1}^{N} \partial_\mu \phi^a \partial_\nu \phi^a \right) - V(\phi^a) \right]$$

$$T_{\mu\nu} = 0 \longrightarrow$$

Composite metric: $$g_{\mu\nu} = \frac{D/2 - 1}{V(\phi^a)} \left(\sum_{a=1}^{N} \partial_\mu \phi^a \partial_\nu \phi^a \right)$$

$$\longrightarrow \quad S = \int d^D x \left(\frac{\frac{D}{2} - 1}{V(\phi^a)} \right)^{\frac{D}{2} - 1} \sqrt{\left| \det \left(\sum_{a=1}^{N} \partial_\mu \phi^a \partial_\nu \phi^a \right) \right|}$$

Mean Field Approximation for Vacuum Spacetime

Batz, JE, Mrini (2021)

Do QFT in presumed vacuum spacetime, insist that $\langle g_{\mu\nu} \rangle = G_{\mu\nu}$

dim reg

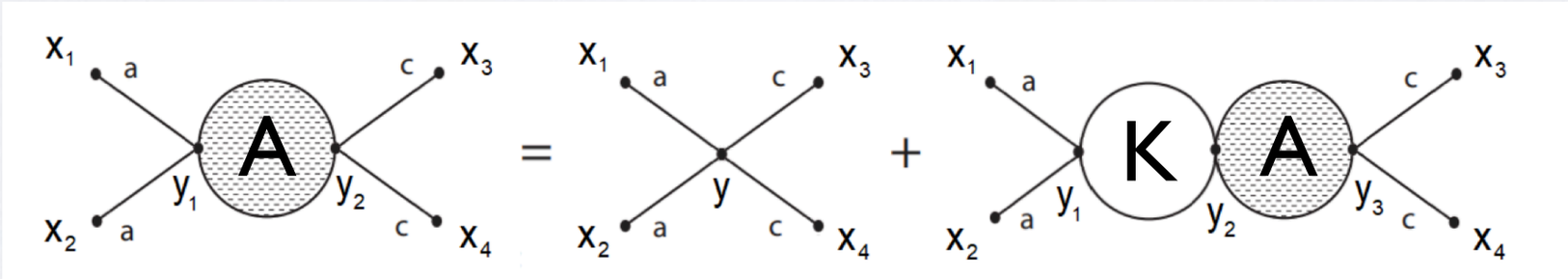
$$\boxed{\frac{1}{(4\pi)^{D/2}} \frac{N}{D-4} \left(\frac{m^{D-2}}{3} \right) \left(R_{\mu\nu} - \tfrac{1}{2} G_{\mu\nu} R \right) = \Lambda G_{\mu\nu}}$$

+ higher-derivative corrections

Emergent gravitational interaction (Batz, JE, Mrini)

$$\langle 0| T\{\phi^a(x_1)\phi^a(x_2)\phi^c(x_3)\phi^c(x_4)e^{i\int \mathrm{d}^D x \mathcal{L}_{\mathrm{int}}(x)}\}|0\rangle =$$
$$\int \mathrm{d}^D y_1 \mathrm{d}^D y_2 \sqrt{|G(y_1)G(y_2)|}\, E_{\mu\nu}(x_1,x_2,y_1) i A^{\mu\nu\alpha\beta}(y_1,y_2) E_{\alpha\beta}(x_3,x_4,y_2)$$

where $E_{\mu\nu}(x_1,x_2,y_1) \equiv \langle \phi^a(x_1)\phi^a(x_2)\mathcal{T}_{\mu\nu}(y_1)\rangle_{\mathrm{con}}$



We find

$$\left[\frac{4\Lambda}{(D-2)}\left(G_{\rho\lambda}G_{\sigma\kappa}-\frac{1}{2}G_{\rho\sigma}G_{\lambda\kappa}\right)-\frac{4M_P^{D-2}}{(D-2)}D_{\rho\sigma\lambda\kappa}(y_1)\right]iA^{\lambda\kappa\alpha\beta}(y_1,y_2)$$
$$=\frac{1}{\sqrt{|G(y_1)|}}\,\frac{1}{2}\left(\delta^\alpha_\rho\delta^\beta_\sigma+\delta^\beta_\rho\delta^\alpha_\sigma\right)\delta^{(D)}(y_2-y_1).$$

where

$$D_{\mu\nu\alpha\beta}\equiv\frac{1}{2}\Big(G_{\mu\alpha}G_{\nu\beta}\Box-\frac{1}{2}G_{\mu\nu}G_{\alpha\beta}\Box+R_{\mu\alpha}G_{\nu\beta}+R_{\nu\alpha}G_{\mu\beta}$$
$$-2G_{\mu\nu}R_{\alpha\beta}+2R_{\mu\nu\alpha\beta}-RG_{\mu\alpha}G_{\nu\beta}+\frac{1}{2}RG_{\mu\nu}G_{\alpha\beta}\Big)$$

This is the propagator equation for a massless spin-two field in the background with cosmological constant, *i.e.* there is gravity!

Stochastic Mechanics/Field Theory

Nelson (1960s);
Guerra, Ruggiero (1970s,1980s)

I will now (try to) convince you that a stochastic description of QFT provides a consistent description of physics at all length scales.

Similar thoughts: Markopoulou, Smolin; Kuipers; Afshordi, Stojkovic

Stochastic quantization (Euclidean) - Parisi, Wu
Stochastic mechanics - Nelson
Stochastic relativistic QFT - Guerra, Ruggiero

Stochastic Differential Equations

$$dx = b_+(x(t), t)\, dt + \sqrt{2D}\, dW(t)$$

Like a velocity

Brownian noise

$$\langle dW(t) \rangle = 0 \qquad dW(t)\, dW(t) = dt$$

$$f(x(t)) = f(x(t_0)) + \frac{\partial f}{\partial x}\, dx + \frac{1}{2}\frac{\partial^2 f}{\partial x^2}\, dx^2 + \ldots$$

Contains term $\propto dt$

Time reverse process

$$dx = b_+(x(t), t)\, dt + \sqrt{2D}\, dW_+(t)$$

step forward in time from *t*

$$dx = b_-(x(t), t)\, dt + \sqrt{2D}\, dW_-(t)$$

step forward from earlier time to *t*

Define $v = \dfrac{b_+ + b_-}{2}$ Convective velocity

$u = \dfrac{b_+ - b_-}{2}$ Stochastic velocity

Forward and Backward Derivatives

$$D_+ f(x,t) = \lim_{\Delta t \to 0} \left\langle \frac{f(x(t+\Delta t)) - f(x(t))}{\Delta t} \right\rangle$$

$$= \frac{\partial f}{\partial t} + b_+ \frac{\partial f}{\partial x} + D \frac{\partial^2 f}{\partial x^2}$$

$$D_- f(x,t) = \lim_{\Delta t \to 0} \left\langle \frac{f(x(t)) - f(x(t-\Delta t))}{\Delta t} \right\rangle$$

$$= \frac{\partial f}{\partial t} + b_- \frac{\partial f}{\partial x} - D \frac{\partial^2 f}{\partial x^2}$$

Stochastic Dynamics

There are inequivalent ways to write F=ma.

For example:

$$F = m\, D_+ b_+$$

$$F = m\, D_+ b_-$$

$$F = \frac{m}{2}(\, D_+ b_- + D_- b_+)$$

Nelson-Newton Eq.

time-reversible

Nelson: The probabilities associated with a stochastic process satisfying the Nelson-Newton equation satisfy the Schrödinger Equation & Born rule.

SHO Ground State

Example - Ornstein-Uhlenbeck process

$$dx = -\omega x\, dt + \sqrt{2D}\, dW_t$$

$$d\langle x^2 \rangle = -2\omega \langle x^2 \rangle\, dt + 2D\, dt$$

Ensemble average

$$\langle x^2 \rangle = C\, e^{-2\omega t} + \frac{D}{\omega}$$

Solution:

$$\xrightarrow{t \to \infty} \frac{D}{\omega}$$

Equilibrium

What about relativistic quantum field theory?

(Guerra, Ruggiero 1970s, 80s)

For free bosonic fields this story is easy to generalize.

Just treat each momentum mode as a harmonic oscillator with the corresponding (state-dependent) stochastic process.

$$\boxed{d\phi(x,t) = -\sqrt{-\nabla^2 + m^2}\,\phi(x,t)\,dt + dW(x,t)}$$

Ground state of free scalar field.

Here, $dW(x,t)\,dW(x',t) = \delta(x-x')dt$

Stochastic Composite Metric

Suppose the free scalar ground state is an approximation to the actual ground state.

$$d\phi(x,t) = -\sqrt{-\nabla^2 + m^2}\,\phi(x,t)\,dt + dW(x,t)$$

Evaluate the composite metric on solution for the scalar fields with arbitrary initial conditions.

$$g_{\mu\nu} = \frac{1}{V(\phi^a)}\left(\sum_{a=1}^{N} D_\mu\phi^a\, D_\nu\phi^a\right)$$

$$(D_0\phi^a)^2 = \frac{1}{2}\left[(D_+\phi^a)^2 + (D_-\phi^a)^2\right]$$

Stochastic Action

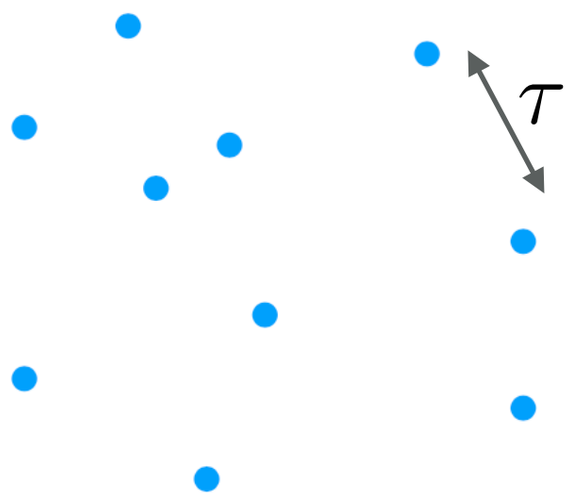
$$S = \int d^4x \, \frac{\sqrt{\det \left| \frac{1}{2} \left[(D_{+\,\mu}\phi)(D_{+\,\nu}\phi) + (D_{-\,\mu}\phi)(D_{-\,\nu}\phi) \right] \right|}}{V(\phi)}$$

Stationarize (Yasue)

Stochastic Equation of Motion

$$\frac{1}{2\sqrt{g}} \left[D_{+\,\mu} \left(\sqrt{g} g^{\mu\nu} D_{-\,\nu}\phi \right) + D_{-\,\mu} \left(\sqrt{g} g^{\mu\nu} D_{+\,\nu}\phi \right) \right] - \frac{\partial V(\phi)}{\partial \phi} = 0$$

Poisson sprinkling of stochastic events (*c.f.* Afshordi, Stojkovic)



Replace dW_t with discrete fluctuations.

There is some fixed probability per unit 4-volume to find a stochastic fluctuation in a small 4-volume.

***c.f.* causal set theory**

A single Poisson sprinkling respects Lorentz invariance, but not Euclidean invariance (Bombelli, Henson, Sorkin)
— selects Lorentzian-signature spacetimes!

Discreteness of stochastic events acts as a regulator — a *physical* regulator that defines the ultra-short distance physics.

Background metric

$$g_{\mu\nu} \overset{t\to\infty}{\to} \int \frac{d^3k}{(2\pi)^3} \frac{k_\mu k_\nu}{V_0\,\omega_{\mathbf{k}}} D(\mathbf{k})$$

$$D(\mathbf{k}) \sim e^{-\omega_{\mathbf{k}}\tau}$$

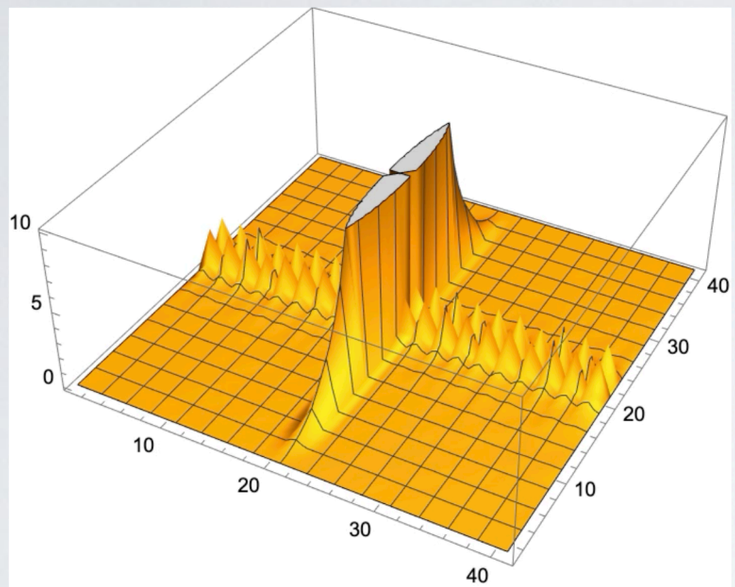
Acts like a ~~Euclidean~~ Lorentzian point-splitting regulator

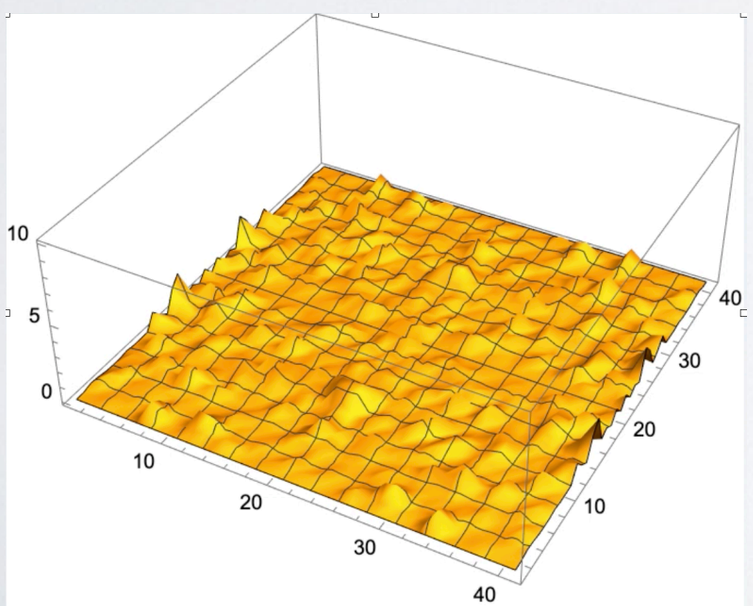
But QFT correlators are Wick rotations of stochastic correlators (Guerra, Ruggiero)

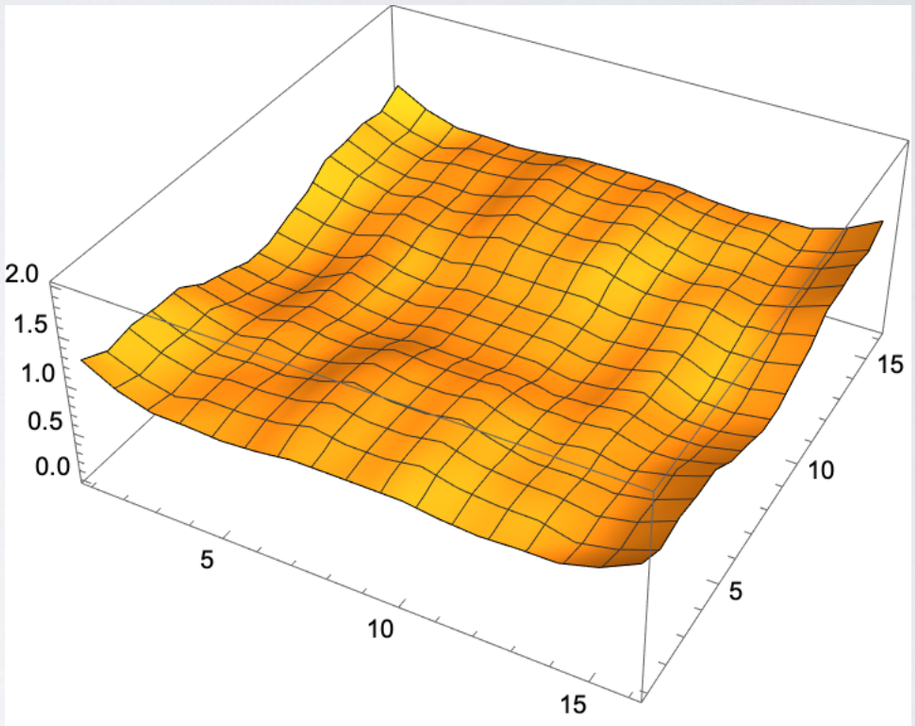
Background metric:

$$g_{xx}(x, y)$$

Arbitrary initial configuration







Average over several Planck volumes

Summary

- Sub-Planckian distances: Classical field evolution with nonstandard dynamics
- Planckian distances: Random fluctuations of fields
- Large distances: Ordinary QFT (after stochastic process has approached equilibrium)

- Sub-Planckian distances: No spacetime, no gravity
- Planckian distances: Random fluctuations act on composite metric (*c.f.* Wheeler's spacetime foam)
- Large distances: Semiclassical general relativity + corrections

Only semiclassical gravity — evades quantum gravity puzzles.

Testability? The stochastic system can be out-of-equilibrium.
c.f. Valentini — cosmology with Bohmian fields.